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THE Franklin Institute is not responsible, as a body, for the statements and opinions advanced by contributors to the JOURNAL.

Franklin Institute.

The Institute has decided to purchase, for experimental purposes, a Dynamo-Electric Machine, capable of producing not less than 1200 candles' light. In order to decide what kind of machine to select, it has been determined to invite all builders of such machines to send one of their make to be submitted to a comparative trial.

The trial will be conducted by a committee of scientific men, well qualified to judge of such apparatus. It is expected that their report will embody a description of each machine, a statement of its performance, including the amount of light produced, power required to drive it, and its capability of long continued work without undue heating or derangement.

It is proposed to make these tests early in October, but it is desirable to know at as early a day as possible, what machines will compete. Those entering their machines for this test, will be expected to send them at their own risk and cost, but no expense will accrue in making the tests. It is believed that such a comparative test as is proposed, and the report thereon, will be of great benefit

by increasing the already great interest felt in this country in the introduction of such machines for producing light, and for other industrial purposes.

Communications on this subject should be addressed to the Secretary.

The following donations to the library were reported at the meeting of the Board of Managers, held August 1st, 1877:

Historical Sketch of Schools in Paterson, N. J. By Wm. Nelson. Paterson, 1877. From the Author.

Royal Geographical Society African Exploration Fund. From the Royal Geographical Society.

Almanac Nautico para 1878 media 1877.

Three Photographs of the Arctic Exhibits made by the U. S. Gov't at the International Exhibition, Philadelphia, 1876.

From the U. S. Naval Observatory.

British Patent-Office Publications, as follows:

Specifications from 1001 to 3200. 1876.

Alphabetical Index of Patentees, etc., for 1874–5. 2 vols.

Abridgments of specifications relating to Bleaching, Dyeing, etc. Part 2, 1858–1866, 2d Ed. 1 vol.

Ice-Making Machines, Ice Safes, etc., 1819–1866. 2 vols.

Manure, Part 2, 1856–1866. 1 vol.

Paper, Pasteboard, etc., Part 2. 1 vol.

Printing, 1617–1857. 1 vol.

Roads and Ways, 1619–1866. 1 vol.

Unfermented Beverages, etc., 1774–1866. 2 vols.

Thirteen Patent Specifications, of odd numbers, to fill deficiencies.

Four numbers Commissioner of Patent's Journal, to fill deficiencies.

Nine Disclaimers. From the Commissioner of Patents.

The Employment of Mechanical Motors on Tramways. By W. R. Rowan. London, 1877.

Tramways. Parliamentary Report. 1877.

From Frederick Ransom.

Geological survey of the oil lands of Japan. Report of progress for the first year. By B. S. Lyman. Tokei, 1877. From K. S. Otori.

Thirty-fourth annual report of the Managers of State Lunatic Asylum, Utica, N. Y., for 1876. From the Managers.

Second report from the Committee on Sewage of Towns. London, 1862.

Transactions of the Society of Engineers, for 1876. Edited by P. F. Murray. London, 1877. From the Society.

Quarterly weather report of the Meteorological Office, Part 4, Oct. to Dec., 1876. London, 1877. From the Committee.

Notice necrologique sur Emile Vandrey par MM. H. Rousselle et J. Darcel. From the publisher of Annales des Ponts et Chaussées.

Les Marques de Fabrique Françaises. Paris, 1877. From G. Morgan Eldridge.

Annales des Ponts et Chaussées. Table generales, 5me ser., 1871–1875. Paris, 1876. From M. Dunod, Paris.

J. B. KNIGHT, *Secretary.*

WATER PRESSURE BLOWING ENGINE.

Editors Journal of the Franklin Institute:

We feel compelled to correct the criticisms of Mr. Briggs, on the Longdale Water Blowing Engine, built by the I. P. Morris Co.

First, The water piston and blowing piston are attached to the same rod. The power is transmitted direct from the one piston to the other, except the small excess at the beginning of the stroke, which is transmitted to the opposite engine and utilized.

Second, The power being transmitted direct, there is but little wear on the journals.

Third, The fly-wheels are mere skeletons, light, and counterbalanced to perform their proper work.

Fourth, The cranks determine the stroke exactly, giving a minimum of clearance in the blowing cylinder, and are always to be relied on.

The Worthington duplex-valve movement has been tried, and does not give a uniform stroke. It is liable to stoppages, which prove fatal to parts of the furnace. Flowing water cannot be suddenly stopped without recoil and straining of the containing parts, hence the use of air chambers.

The *When* is given correctly by Mr. Briggs.

The *How* is in error.

The *Why* he has not examined.

The Longdale engines are working economically and satisfactorily, without Davey regulators or governors.

TAWS AND HARTMAN.

"THE GAMUT OF LIGHT."

By Pliny Earle Chase.

In accordance with a suggestion of Dr. Henry Draper, that I should test some of my views by applying them to the spectral lines, I have undertaken a preliminary investigation, with the following result:

In the harmonic progression, $\frac{c}{n}, \frac{c}{n+a}, \frac{c}{n+2a}$, etc., let c = wave-length of Fraunhofer line $A = 761{\cdot}20$ millionths of a millimetre; $n = 1{\cdot}0150$; $a = {\cdot}0918$; and we find the following accordances:

Numerator.	Divisors.	Quotients.	Observed.	Kirchhoff Lines.
761·20	n	749·95	?	?
	$n + a$	687·75	687·49 *B*	592·7
	$n + 2a$	635·07	634·05	783·8
	$n + 3a$	589·89	589·74 $D\gamma$	1005·8
	$n + 4a$	550·72	550·70	1306·7
	$n + 5a$	516·42	517·15	1655·6
	$n + 6a$	486·14	486·52 *F*	2080·0
	$n + 7a$	459·22	458·66	2436·5
	$n + 8a$	435·12	435·67	2775·7
	$n + 9a$	413·43	?	?
	$n + 10a$	393·79	393·59 *H′*	*H′*

The "observed" values are the wave-lengths, as determined by Dr. Wolcott Gibbs (*Amer. Jour. Sci.* [2], xliii, 4), of the corresponding lines on Kirchhoff's scale. The lines between *A* and *B* have not been studied sufficiently to fix their wave-lengths; it seems likely that $A \div n$, may be a *bright* line. The line $A \div (n + 9a)$ is midway between Kirchhoff line 2869·7 = 430·37, and $H = 397{\cdot}16$.

As in the solar system alternate planets, so in the Fraunhofer lines, alternate lines appear to obey the simplest law, the intermediate lines being governed by laws of mutual equilibrium. The figurate symmetry of the divisor differences for the alternate lines *B*, *D*, *F*, *H* (1 *a*, 3 *a*, 6 *a*, 10 *a*), is specially noticeable, and suggestive of my equation between the principal planetary masses:

$$(\text{Neptune})^{1} \times (\text{Uranus})^{3} \times (\text{Jupiter})^{6} \times (\text{Saturn})^{-10} = 1.$$

Haverford College, *Aug.* 23*d*, 1877.

SATELLITES OF MARS.

The following is an extract from the letter of Rear Admiral John Rodgers, Superintendent of the U. S. Naval Observatory at Washington, to the Secretary of the Navy, dated August 21st, 1877, announcing the discovery of two satellites of the planet Mars:

"The outer satellite of Mars was first observed by Professor Asaph Hall, U. S. N., on the night of the 11th of August, 1877. Cloudy weather prevented the certain recognition of its true character at that time. On August 16th it was again observed, and its motion was established by observation extending through an interval of two hours, during which the planet moved over thirty seconds of arc.

"The inner satellite was first observed on the night of August 17th, and was also discovered by Professor Hall.

"On Saturday, August 18th, the discoveries were telegraphed to Alvan Clark and sons, Cambridgeport, Mass., in order, that if the weather should be cloudy at Washington, they might confirm the existence of the satellites with the 26-inch telescope of Mr. McCormick, which is in their hands.

"The discovery was confirmed by Professor Pickering and his assistants, at Cambridge, Mass., and by the Messrs. Clark, at Cambridgeport.

"On August 19th the discovery was communicated to the Smithsonian Institution, by which it was announced to the American and European observatories * * * *."

Stability of English Iron-Clads.—It will be remembered that among the late experiments in naval architecture by the British Government, has been the construction of what are called citadel ships. These ships have the central portion of the hull, for about one-third its length, enclosed with armor of immense thickness, for the protection of the machinery, magazine and guns; the remaining portions being unarmored.

Of these, the *Inflexible*, supposed to be the most powerful vessel ever designed for the British navy, measures 320 ft. in length, 75 ft. in breadth at the water-line, with a mean draught of water of 24 ft. 5 in., and a total displacement of 11,407 tons. The central citadel is 110 ft. long by 75 ft. wide, and is protected with armor-plates of a total

thickness of 24 in. from 6 ft. below the water-line to 10 ft. above. Forward and aft of the citadel the hull of the vessel is entirely unprotected, except by a 3 in. armor deck 7 ft. below the water-line.

The *Inflexible* is now approaching completion, and two others on the same principle, the *Ajax* and *Agamemnon*, have been commenced, at an estimated cost of £500,000 each.

The London *Times*, of June 18th last, made the startling announcement that notwithstanding the 24 ins. of armor with which the *Inflexible* is protected, she will be capable of being sunk by shell-fire from an enemy, without her armor being even touched. This statement is based upon the alleged fact that the armored citadel does not provide the requisite stability to enable her to float upright when the unarmored ends are practically destroyed.

Mr. E. J. Reed, formerly chief constructor in the navy, and now a member of Parliament, repeated these charges in the House of Commons, in the evening of the same day, and stated that, from careful calculations made by himself, it was certain that the *Inflexible* would have no stability whatever, should the unarmored ends be destroyed.

Mr. Ward Hunt, First Lord of the Admiralty, denied these statements, and asserted most positively, on the strength of a recent inquiry into the matter, that she would have the requisite stability if the unarmored ends were completely perforated.

In the discussion which has since taken place, the stability of other armored vessels is called in question, and the position taken by Mr. Reed causes the more anxiety when it is remembered that just before the loss of the *Captain*, the Admiralty reported that she would be safe even with such damage as she was likely to receive in action, and yet she capsized while perfectly intact, and in an ordinary sea-way.

Such charges, involving great incompetency on the part of the Admiralty Board, were of too grave a character to be lightly passed over, and the subject was referred by the House of Commons to a committee of experts, whose report will be looked for with much interest by those interested in naval architecture. K.

Gout.—Dr. Th. Max. Sorel describes a successful treatment of hereditary gout, in his own case, by digitalis, quinine, and "Palmerston pills," with careful attention to exercise and diet.—*Les Mondes.* C.

Widening Delaware Avenue.—A commission, consisting of Messrs. Strickland Kneass, D. Hudson Shedaker, Robt. Briggs and Samuel L. Smedley, was appointed under a resolution of Councils of this city, on May 11th last, to make a survey and estimate the cost of widening Delaware Avenue, between Walnut and Vine Streets.

The object of this proposed widening was to make room for a line of railroad tracks connecting the lines of the Pennsylvania Railroad and those of the Reading Railroad, for the better accommodation of the traffic along that part of the Delaware river front.

The Commission has reported, and recommends that the width of Delaware Avenue be fixed at 80 ft. instead of 90 ft., as proposed by Councils, and that it shall be widened entirely on the eastern or river side, by appropriating 30 ft. of the heads of docks and shore ends of wharves.

The Commission believes that by a re-arrangement of some of the wharves or piers, and the extension of others to the Port Warden's line, the commerce now using this portion of the river front, with few exceptions, can be more fully provided for than at present.

They estimate that the cost of carrying out this plan, inclusive of any reasonable amount for property appropriated therefor, will, at present prices, not exceed $470,000. K.

A Finland Volcano.—Vast masses of smoke are reported as issuing from a mountain near the river Tara, and the snow in the neighborhood has been melted. The gradual elevation of the Bothnian shores has often been attributed to volcanic forces, and it is possible that they are finally seeking a vent. The region has, hitherto, been free from positive evidences of volcanic activity.—*Nature.* C.

Galvanic Crystallization.—The journal of the Russian Chemical and Physical Society, vol. ix, fas. 2, contains observations, by Shidlovsky, on the microscopical crystallization of various metals under the influence of a galvanic current. The dondritric agglomerations of crystals form very speedily; their branches spread out from the cathode to the anode plate, vibrate on reaching it, and collapse; this process is repeated till the space between the plates is filled with a spongy metallic mass. Each metal has a characteristic ramification. The crystallization does not appear when the anode is gold or platinum.—*Nature.* C.

Endowment of Research.—The British Government, on the recommendation of the Royal Society, has appropriated £3935 in aid of scientific research during the present year. The largest appropriations are of £300 each, to Prof. Parker, for morphological researches; to Dr. H. E. Armstrong, for researches into the Phenol Series and the effect of nitric acid on metals; and to W. Crookes, for researches connected with repulsion resulting from radiation.—*Nature.*

C.

The Jablochkoff Light.—In late Parisian experiments the kaolin bar, ignited by induction spark, was 8 centimetres long, and fully equal to 8 gas-burners. At the same time 3 electric candles were operated, each equaling about 40 gas-burners. The light is admirable for constancy and duration. The thickness of the kaolin plate is not more than 4 millimetres, and the quantity consumed is not more than 1 mm. per hour. The agitation of the candlestick does not interrupt the current. The graphite pencils are consumed at the rate of 8 centimetres per hour, but it is thought that any length required may be supplied by proper clockwork.—*Nature.* C.

Vegetable Ferment.—M. C. Kosmann cuts plants into small bits, which, when macerated in cold water, evaporated, and placed in thrice their volume of alcohol, give a white precipitate which is redissolved in a small quantity of water; after filtration it is precipitated anew in a triple volume of alcohol, and a ferment collected, which, when dry, is in the form of translucent amorphous grains, brownish, insipid, soluble in water, decomposable by boiling, and containing nitrogen. This ferment seems to be a general principle common to all plants, which plays an important part in the formation of proximate principles and their metamorphoses in the interior of the organized tissue. Kosmann regards it as a universal motor in the vegetable kingdom, which is developed at the same time as the cell, and which is probably secreted by the protoplasm. The chemical disturbance which arises on the decomposition of the ferment, and which is communicated to the starch, sugar, and glucosides, is not a fact peculiar to the ferment; the same thing occurs in the chemical action consequent upon the oxidation of iron in contact with air and water. Metallic iron, in oxidizing, converts starch into dextrine, glucose, or butyric acid; moist iron, oxidizing at the surface of water, stimulates the decomposition of starch.—*Les Mondes; Soc. de Chim. de Paris.* C.

Vacuum Brake.—The engineers of the French Northern Railway have made satisfactory experiments with the vacuum brake. MM. Sartiaux and Lartique have devised some ingenious arrangements for bringing it into automatic action, if any mistake is made respecting the crossings. Distressed passengers, who need help, can operate the brake instead of ringing a bell.—*Nature.* C.

An African Sea.—A writer in the *Scientific American* has lately estimated the amount of evaporation from the surface of the proposed Algerian Sea, the reduction of the ocean level, and the time required to fill the sea with a bed of salt. In his calculations he seems to have omitted any consideration of the amount of vapor that would be recondensed, over the sea or in the neighborhood of its banks, so as to find its way back into the reservoir from which it was drawn. The elements involved in the question are too numerous to be determined by a few ingenious numerical calculations, and the discussions in the French Academy show that there is no lack of able engineers and physicists, who are thoroughly competent and willing to look at the project in all its climatic and economical bearings, before any final decision is made.—*Comptes Rendus.* C.

Recomposition at Temperatures Higher than those of Dissociation.—It is well known that most bodies are decomposed under the influence of heat, and that if the temperature is high enough, their decomposition is complete. It seems natural to suppose, that above this temperature the compounds could not exist. But we have demonstrated, by the study of many compounds of silicium, that this conclusion is too general. 1. The sesquichloride of silicium, in particular, which is very stable at ordinary temperatures, begins to decompose at about 350° C.; its decomposition is complete at about 800°. It may be represented by the formula, $2\,Si_2\,Cl_3 = 3\,Si\,Cl_2 + Si$. The sesquichloride is formed again, if we place the products of its decomposition in a porcelain tube, at about 1200°: $3\,Si\,Cl_2 + Si = 2\,Si_2\,Cl_3$. The sesquichloride thus formed may be isolated by rapid cooling. If it is brought gradually into parts of the tube where the temperature does not exceed 800°, it is decomposed, and gives crystals of silicium which obstruct the tube. In this case we get bichloride, which boils at 58°, instead of sesquichloride, boiling at 146°. The sesquichloride of silicium presents, therefore, a great stability at a temperature much higher, as well as at a temperature much lower, than that which determines its

complete decomposition. We have observed similar phenomena with the protochloride and the subfluoride of silicium. These facts are not exceptional. M. Ditté has proved that selenhydric and tellurhydric acids, which are easily decomposed by heat, may be reproduced from their elements at a temperature higher than that of decomposition. 2. Platinum, heated to about 1400°, is neither fusible nor volatile in an atmosphere of nitrogen, oxygen or hydrogen; but if it is heated in a porcelain tube, and some bubbles of chlorine are admitted, small crystals of platinum are deposited in the parts of the tube which are at a lower temperature. The metal, therefore, acts as if it were volatile in the chlorine. This apparent volatilization is the result of the decomposition, by reduction of temperature, of a chloride of platinum formed at a very high temperature. To secure this chloride, we inserted in the porcelain tube a slender tube of glass, which was kept cool by a current of water. The compound, which was deposited on the lower part of the cold tube, proved, upon analysis, to be protochloride of platinum. 3. It is well known that ozone passes into the condition of ordinary oxygen at about 250° C., but if a tube containing oxygen, at a temperature of 1300° to 1400°, is traversed by a silver tube which is kept cool by a current of water, the surface of this tube is coated with binoxide of silver—insoluble in acetic acid; soluble, with liberation of gas, in ammonia. This is exactly what would have happened at the ordinary temperature, with oxygen ozonized by the known methods. If, by a tube of small diameter lodged in the cold tube, we extract the oxygen which has been ozonized by the heat and rapidly cooled, we can produce the discoloration of indigo and the characteristic variations of ozone; but if the ozonized oxygen is gradually cooled, it undergoes a complete decomposition, and we collect only ordinary oxygen. 4. Proust observed, that with the common blowpipe, silver gives a coating containing a little oxide of silver. H. Ste.-Claire Deville and Debray have shown that, by rapidly cooling the vapor of silver which is boiling in contact with the air, metallic silver may be obtained, mixed with a small quantity of oxide of silver. We may ask whether the oxide of silver, a body easily decomposed by heat, was really produced at a high temperature, or whether it resulted from a reaction between the cold silver and the hot ozonized oxygen. To settle the question, we vaporized silver in a porcelain tube, heated to 1400° and traversed by a tube which was well cooled by a current of water.

We thus collected, upon the cold tube, metallic silver mixed with a large proportion of protoxide of silver. In the preliminary experiments with the same tube, we proved that cold silver, in contact with very hot oxygen, gives only binoxide of silver, without a trace of protoxide. Therefore, protoxide of silver, although decomposed at a moderate temperature, may be produced at a very high temperature, like the other bodies that we have been studying.—MM. Troost and Hautefeuille, in *Comptes Rendus*, April 30. C.

Prizes of the French Academy.—The following prizes have been awarded:

1. Grand mathematical prize, to G. Darboux, for his discussion of the theory of singular solutions of equations for partial derivatives of the first order.

2. Extraordinary prize of six thousand francs, for the application of steam to the naval service, to A. Ledieu, for his publications upon ships of war and marine engines, from 1862 to 1876.

3. Poncelet prize to M. Kretz, for his aggregate works, and especially for the intelligent and devoted care which he has given to the publication of Poncelet's works.

4. Montyon mechanical prize, to M. Deprez, for his gravity- and inertia-integrator, and his pressure-indicator.

5. Dalmont prize, to M. Ribaucour, for his geometrical works, and especially for his manuscript memoir upon the theory of surfaces.

6. Lalande prize, to M. Palisa, for the discovery of nine new asteroids in 1874 and 1875, and the re-discovery, in 1876, of the asteroid Maia, which had been lost from observation for fifteen years.

7. Bordin prize, divided as follows: To M. Violle, 2000 francs; to M. Crova, 1000 francs; to M. Vicaire, 1000 francs; for their investigations relative to the temperature of the sun's surface.

8. Montyon statistical prize not awarded, but certificates of honorable mention to Dr. Bertillon, for his atlas, entitled "Demography of France, mortality according to age, sex, civil condition in each department, and for France entire;" to G. Heuzé, for his atlas, entitled "Agricultural France;" to Dr. G. Delaunay, for his manuscript memoir, entitled "Studies upon the civil condition of the commerce of Creil."

9. Jecker prize, to M. Cloez, for his researches relative to the oil of the seeds of *Elæococca vernicia*.

10. Barbier prize, to Prof. Planchon, for his work, in two volumes, upon Materia Medica. Gratuities were also awarded, of 1000 francs,

to MM. Gallois and Hardy, for their studies upon the bark of *Erythrophlœum guineense;* and of 500 francs, to Dr. Lamarre, for his work on the treatment of whooping cough.

11. Desmaziéres prize, to Ed. Bornet, for the "Collection of observations upon the algæ," by Bornet and Thuret, M. Thuret having died since the publication. A gratuity of 500 francs was also awarded to M. Müntz, for studies of mushrooms.

12. Thore prize, of 500 francs, to E. Oustalet, for his researches upon the tertiary fossil insects of France.

13. Bréant gratuities of 2000 francs, to Dr. Duboué, and 1000 francs to Dr. Stanski, for certain specified medical memoirs.

14. Montyon medical and surgical prizes: (1) To MM. Feltz and Ritter, for their clinical and experimental study upon the action of the bile, and of its principles introduced in the organism; (2) to Dr. Paquelin, for his new cautery; (3) to Prof. Perrin, for his treatise of practical ophthalmoscopy and optometry. Several certificates of honorable mention were also awarded.

15. Montyon prize in experimental physiology, to MM. Morat and Toussaint, for their work on the variation in the electric condition of the muscles, in the different forms of contraction. A medal of 500 francs was also awarded to M. Mialhe, for his labors in physiology and biologic chemistry.

16. Montyon prize, concerning unhealthy arts, of 2500 francs, to Prof. Melsens, for his successful employment of iodide of potassium as a specific in mercurial diseases.

17. Tremont prize, to Prof. Ch. André, for valuable experimental representations of the different phases in the transit of Venus.

18. Gegner prize, to M. Gaugain, for his scientific labors, memoirs and inventions, during twenty-five years, especially in electricity and magnetism.

19. Cuvier prize, to M. Fouqué, for his geological studies, in two series—the first having for their object the composition of volatile volcanic emanations; the second, the determination of the constituent minerals in ancient and modern lavas.

20. Delalande-Guerineau prize, to H. Filhol and Ch. Vélain; to the former, for his description of Campbell Island, and for his zoological, botanical and anthropological researches in Polynesia; to the latter, for his geological studies in Europe and in the Southern hemisphere.

21. Marchioness de Laplace prize, of a complete collection of Laplace's works, to Louis-Paul Henriot, first scholar of the Polytechnic School in the year 1876.—*Comptes Rendus*, April 23. C.

Geological Progress.—MM. Delesse and de Lapparent have prepared a valuable résumé of the geological works published during the years 1875 and 1876. Their work covers 184 closely-printed pages; we have room only for a few brief notes. The mean height of Europe, according to Leipoldt, is 296·838 metres; Humboldt's estimate was 205 m. The increase of temperature at given depths below the surface, is greatest in the equatorial regions. Prestwich has confirmed the views of Dana, Carpenter and Wyville-Thomson, relative to the distribution of ocean temperatures. The resistance of rocks to crushing is diminished (in some cases as much as 80 per cent.) by the absorption of water. The plasticity of surface rocks is intimately dependent on their argillaceous character; but at great depths, pressure, water and increased temperature, make all rocks plastic. Th. Hübener has demonstrated, in a lignite, the existence of a multitude of microscopic quartz crystals, which he attributes to a slow decomposition of infiltrated silicates by the humic acid. By treating a Vesuvian pumice, which seemed to be amorphous, with fluorhydric acid, Fonqué has extracted from it crystals of feldspar, pyroxene, amphibole, peridote, magnesian mica, and oxidized iron. He has also shown that the minute cavities of the pumice were decked with microscopic crystals of amphigene. The contest respecting the organic character of the *Eozoon* still continues; and even if its animal origin is granted, doubts are thrown on the assumed age of the Lawrentian formation, in which it is found. Owen has studied the bones of a curious carnivorous reptile, *Cynodrakon major*, from southern Africa. He assigns it, together with other similar reptiles from the same region, to a new order, *Theoriodontes*, having the dentition of carnivores. He thinks that their high organization cannot be explained by the hypotheses, either of Darwin or of Lamarck. Forests improve the soil much more rapidly than coppice-wood; the humus exhibits a very different composition from that of the rocks upon the surface of which it is formed. Experiments with Tresca's apparatus, seem to show that cleavage and lamination may be due to the same cause, and that the schistosity of gneiss may be no evidence of stratification.—*Ann. des Mines*, x, 438, *seq.* C.

ELECTRIC LIGHTING.

The following letter, addressed to the Secretary of the Franklin Institute, in reply to inquiries as to the success attending the use of the Electric Light at the Railway station mentioned, gives some interesting and valuable statistics.

Paris, 11*th July*, 1877.

SIR :—I take pleasure in sending you the information you desire, in regard to our experiments in electric lighting, made at the station of La Chapelle, Paris.

1. The source of electricity is the Gramme machine, of which I enclose a description; those which we have are of the type A. They require a force of 2·9 horse power (75 kgr.); in operation this force is somewhat reduced.

2. Up to this time we have used the regulator of M. Servin, with carbons of gas coke, having a square section, 0·009 m. on the side; the rods, 0·22 m. in length for the positive, and 0·11 m. for the negative, last four hours on the average. We have tried agglomerated carbons; they give a fine light, and are very regular in action, but the consumption is much more rapid, and we are waiting for their manufacture to be perfected. Many inventors are engaged upon them.

We have also tried a new regulator of M. Reynier, working more than 24 hours without change of carbons, but our experiments are not yet sufficient to give us any decided opinion on this new contrivance.

3. Our efforts have been chiefly directed towards obtaining a sufficient light, moderate, as uniform as possible, and not fatiguing the eyes of the employés; we have then sought to avoid the direct action of the luminous rays, and especially to use the reflected or diffused rays; for this purpose the lamp is enclosed in a large lantern, with glasses clouded by zinc white; this clouding is thick enough, and extends high enough, to prevent the voltaic arch being seen when the lantern is in place. We thus substitute, for the dazzling point furnished by the carbon pencils, a large shining body, the action of which cannot injure the retina.

In the halls, the rays which escape by the upper part of the lantern are sent, with all their intensity, towards the ceiling and the

top of the walls, which have been whitened, and serve as diffusing reflectors; we thus obtain, even when employing only a single lamp, placed in the middle of the hall, at a height of five metres, a general light, very suitable for the management and recognition of the packages, without troublesome shadows, and sufficient for a radius of 35 to 40 metres. In the open air, the lantern is closed at the top by a cap of tin plate, which serves much more as a cover than as a reflector. The walls of the lantern are formed of two glasses, with an interval of 0·01 m. between them, so that the outer glass is never heated enough to be broken by a sudden chill, and it protects the inner one.

A lamp, in these conditions, placed at a height of 6 or 7 metres, gives, in a radius of at least 60 to 80 metres, a light amply sufficient for the management of the cars.

The lanterns are prismatic, 0·45 m. to 0·50 m. on a side, and 1 m. to 1·10 m. in height. It would probably be much better if they were cylindric, or better still, pyramidal or inverted cones.

Our three lamps, working 10 hours per day, require:

Motive power,	6·85 f.
Mechanic, 10 h. @ ·50,	5·00 f.
Carbons,	4·50 f.
Interest and sinking fund, at 10 per cent. of the plant, having cost 23,000 f., say $\frac{2300}{365}$,	6·30 f.
Oil, repairs, cleaning,	1·00 f.
Total,	23·65 f.

one-third of which is 7·90 f. for ten hours' work, or ·79 f. per lamp per hour.

Preparation has been made for five lamps; in other words, the steam engine, and the building constructed expressly to shelter the machine, are sufficient to operate five Gramme machines. The expense would then be:

Motive power,	9·05 f.
Mechanic,	5·00 f.
Carbons,	7·50 f.
Care and cleaning,	1·50 f.
Interest and sinking fund at 10 per cent. of the capital, $\frac{2300}{365}$,	7·50 f.
Total,	30·55 f.

or about 0·60 f. per lamp per hour.

The sinking fund of 10 per cent. should be reduced to 8 per cent., on account of the buildings and the Gramme machines, which may be counted at 6 per cent. If we had not erected a building, the expense would have been lessened by more than 4000 f. The Gramme machines cost in Paris, 1500 f. apiece, the lamps 450 f., but I allow for two in the price of the plant. The lanterns, with their pullies, etc., cost 100 francs.

I beg you, Sir, to accept the assurance of my great respect.

For the Engineer-in-Chief,

A. SARTIAUX, *Asst. Eng.*

Book Notice.

THE ECONOMIC THEORY OF THE LOCATION OF RAILWAYS. By Arthur M. Wellington, C.E. 12 mo, pp. 230, cloth, $2.00. Railroad Gazette, N. Y., 1877.

This is an exceedingly practical and valuable work.

The error of most engineers has been to accept certain rules laid down by the earlier engineers as infallible, and apply them to all cases. Whereas each line of road presents problems to be solved by the data offered by its peculiar circumstances and conditions, and which may not be applicable to any other.

Time has been lost, capital wasted, and conclusions reached, and the efficiency of practical operation impaired by attempting to apply antiquated rules in regard to equation of distances, and curvatures, gradients, slopes, etc.

Mr. Wellington seems to have comprehended the true conditions, and has given sound, practical solutions of those economic problems on which the location of a line depends, and which perpetually affect its cost of operation. The book is needed. Had it appeared earlier, many engineering blunders might have been avoided, capital saved in construction, and dividends increased by reducing expense of operation.

We learn that the book will be used in the course of study in the department of civil engineering at the University of Pennsylvania.

H.

"THE MUSIC OF THE SPHERES."

By Pliny Earle Chase, LL. D.

More than three thousand years are supposed to have elapsed since Job heard the voice out of the whirlwind, discoursing of the time "when the morning stars sang together." It is nearly twenty-five hundred years since Pythagoras traveled into Egypt and the East, to be initiated into the mysteries of the Persian and the Chaldean Magi, the Indian Gymnosophists, and the priests of Amun-Ra. It was in these journeys, that he probably learned the doctrines of musical harmony, which became the groundwork of his own philosophy.

The disciples of Pythagoras claimed that he was the only mortal whom the gods had permitted to hear the music of the spheres. He taught that the orbits, in which the planets move, dividing the ether in their course, produced tones; and that the tones differed according to the size, velocity, and distance of the planets. In consequence of his notions of the supreme perfection of the universe, he necessarily believed that these relations were in concord, and produced the most perfect harmony.

About twenty-one hundred years after the death of Pythagoras, John Kepler made the first practical demonstration of the harmonies which Job had learned so long before. His most important results were embodied in the three famous laws, on which Newton's subsequent discoveries, and the whole modern theory of planetary motions, are founded. These laws may be thus expressed: 1. Every planet revolves in an ellipse, of which the sun occupies one of the foci. 2. The radius vector of any planet describes equal areas in equal times. 3. The squares of the times of planetary revolution are proportioned to the cubes of their mean distances from the sun. In announcing his discovery, he says: "Nothing can restrain me; I yield to the sacred frenzy. I dare ingenuously to confess, that I have stolen the golden vessels of the Egyptians, and will build of them a tabernacle to my God. If you pardon me, I rejoice; if you reproach me, I can endure it; the die is thrown. I write a book to be read; whether by the present, or future ages, it matters not. It can wait a century for a reader, if God Himself waited six thousand years for an observer."

Kepler also noticed the regularity in the increase of planetary intervals, in proceeding towards the outer limits of the solar system. Prof. Titius, of Wittenberg, expressed that regularity in the form of an empirical law, which Prof. Bode, of Berlin, subsequently simplified, and consequently published a suggestion, that a planet, which had not yet been discovered, might be found between Mars and Jupiter. This happy anticipation was verified on the first day of the nineteenth century, when Piazzi, at Palermo, discovered Ceres. Prof. Harding, of Göttingen, discovered Juno in 1804; Pallas and Vesta were both discovered by Dr. Olbers, of Bremen, in 1802 and 1807 respectively. In 1845, a new series of discoveries began, in the same belt, and now nearly 200 asteroids are known, whose mean distance is in striking accordance with harmonic prediction. The agreement of planetary positions with "Bode's Law" is shown in the following table:

	Mercury.	Venus.	Earth.	Mars.	Pallas.	Jupiter.	Saturn.	Uranus.	Neptune.
Theoretical . .	4	7	10	16	28	52	100	196	292
Observed . . .	3·9	7	10	16·4	27·7	52	100	191·8	296

The numbers in the lower line represent the mean aphelion of Mars and Saturn, the secular perihelion of Neptune, the mean perihelion of Venus, and the mean distances of the other planets. The law, as stated by Bode, provided for an indefinite continuance of the doubling intervals. Accordingly, when the discovery of Neptune confirmed the predictions of Adams and Leverrier, its distance was supposed to be 388, and when it was found to be only about three-fourths as great, Bode's law was said to have failed. But a glance at the table will show that it was merely modified, by the introduction of a new symmetry, for there are now two equal intervals at each extremity, the intervening intervals being uniformly doubled as we go outward, or halved as we go inward. The present indications, therefore, make it probable that Mercury and Neptune are the boundaries of proper planetary aggregation, although there may be asteroidal, cometary, fragmentary, and temporary clusters outside of each of those boundaries. This probability is strengthened by a variety of other considerations, one of the most important of which will be given in explaining the probable origin of this special harmony.

In 1849, Prof. Peirce communicated to the American Association for the Advancement of Science, the results of an investigation, made by Thomas Hill,[1] at Peirce's request, showing that the phyllotactic

[1] Subsequently President of Harvard University.

series, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{13}$, could be found in the orbital times of planets, as well as in the arrangement of buds and other organs of plant growth. Each term of this series, after the second, is formed by adding the numerators of the two preceding terms for a new numerator, and the denominators for a new denominator. Chauncey Wright, in *Runkle's Mathematical Monthly*, referred the phyllotactic law to nodes of extreme and mean ratio, which he called "the *distributive ratio*." In its application to plants, it distributes leaves most evenly around the stem, so as to provide for the most general participation in the beneficent influences of light, air, moisture, and vegetable nutriment; in its application to planets, it distributes all their mutual perturbations most evenly around the sun, so as to contribute to the stability of the system. These are, perhaps, the two most striking mathematical demonstrations that have ever been brought forward, to show the universal sway of Intelligence and Design.

The approximate phyllotactic ratios of orbital times are: Uranus $= \frac{1}{2}$ Neptune, Saturn $= \frac{1}{3}$ Uranus, Jupiter $= \frac{2}{5}$ Saturn, Asteroid $= \frac{3}{8}$ Jupiter, Mars $= \frac{5}{13}$ Asteroid, Earth $= \frac{1}{2}$ Mars, Venus $= \frac{8}{13}$ Earth,[1] Mercury $= \frac{2}{5}$ Venus. By taking the mean distances which would represent these phyllotactic times, we find the following accordance:

	Mercury.	Venus.	Earth.	Mars.	Pallas.	Jupiter.	Saturn.	Uranus.	Neptune.
Theoretical, . .	3·7	6·7	9·3	14·8	28·	53·8	99·1	206·	327·1
Observed, . . .	3·9	6·7	9·3	15·2	27·7	54·3	100·	206·8	304·7

The numbers in the lower line represent the secular aphelia of Neptune and Uranus, the mean aphelia of Saturn and Jupiter, the secular perihelia of Earth and Venus, and the mean distances of Mars and Mercury. The upper numbers are those which would give the theoretical time-ratios, and would also yield a precise agreement between the theoretical and observed geometrical mean-times. The unit of comparison, both in this and in the previous table, is $\frac{1}{10}$ of Earth's mean distance from the Sun.

In 1850, Prof. Alexander presented to the American Association the first of a series of papers upon harmonies of the solar system. His final results were embodied in No. 280 of the quarto Smithsonian Contributions, published in March, 1875. He has shown that the relative mean distances of the largest planets, Jupiter and Saturn, are such as to have given them equal moments of inertia when the solar system was condensing from the primitive nebula, and that the

[1] $\frac{8}{13} = 1 - \frac{5}{13}$.

consecutive ratios of other planetary distances can be closely represented by the $\frac{1}{2}$, the $\frac{2}{3}$, or the first power of the Jupiter-Saturn ratio. The following table shows the closeness of his approximations:

	Mercury.	Venus.	Earth.	Mars.	Pallas.	Jupiter.	Saturn.	Uranus.	Neptune.
Theoretical, . . .	3·9	7·3	9·9	15·7	28·8	52·3	94·5	195·6	300·6
Observed, . . .	3·9	7·2	10·	15·2	27·7	52·	95·4	191·8	300·6

The numbers in the lower line all represent mean distances.

In 1871, Prof. Kirkwood sent a paper to the American Philosophical Society, in which he says[1]: "It is a very remarkable fact, in regard to the systems of both primary and secondary planets, that the periods, without any exception, have very simple relations of approximate commensurability." His theoretical arrangement of the planetary times differs from that of Prof. Peirce only at two points. Kirkwood adopts for Earth, $\frac{1}{2} \times \frac{1}{3}$ Jupiter, instead of $\frac{3}{5} \times \frac{5}{13}$ Jupiter, and for Venus, $\frac{2}{3}$ Earth, instead of $\frac{8}{13}$ Earth. Therefore, his ratios, as well as Peirce's, are all phyllotactic. Taking corresponding mean distances, as in Peirce's comparative table, and using an extreme instead of a mean asteroidal term, we get the following table:

	Mercury.	Venus.	Earth.	Mars.	Hygeia.	Jupiter.	Saturn.	Uranus.	Neptune.
Theoretical, . . .	3·9	7·3	9·5	15·1	31·4	49·8	91·8	191·	303·1
Observed, . . .	3·9	7·2	9·7	15·2	31·5	49·8	90·8	192·	303·4

The numbers in the lower line represent the mean aphelion of Neptune, the mean perihelia of Saturn, Jupiter and Earth, and the mean distances of the other planets.

In 1873, the writer communicated to the American Philosophical Society[2] some comparisons between different planetary series, together with a new harmonic series of his own, and the following hypothetical explanation of Bode's law: "If we conceive a rotating nebulous mass with a slight equatorial nucleus of condensation, the line of particles between the surface and the centre will be influenced by tendencies to two different kinds of motion: first, as portions of a rotating mass, with velocities varying as the distance; second, as revolving particles, with velocities varying inversely as the square root of the distance. The first of these tendencies, combined with the moment of inertia, would urge such particles as were free to move, towards the linear centre of oscillation. If Mercury's mean distance be taken as the point of suspension, Uranus is situated

[1] Proc. Soc. Phil. Amer., xii, 163.

[2] Ibid, xiii, 470 *seq.*

approximately at Neptune's linear centre of oscillation, and each of the planets between Uranus and Venus is at the linear centre of oscillation between the next superior and the next inferior planet."

According to the calculations of Sir William Thomson, the precession of the equinoxes requires a theoretical rigidity in the Earth greater than that of steel. Such a rigidity is, of course, entirely opposed to anything that we know, as well as to everything that we can easily conceive, of the earth's internal structure. But the modern discoveries relative to the importance of æthereal undulations, suggest the possibility that the rigidity, which is theoretically requisite, may be found in those undulations. The wave velocity, in elastic media, varies as the square root of the quotient of the elasticity, by the density. The relative rigidity, therefore, if we measure it by the ratio of elasticity to density, is $(183{,}000 \div 3)^2 = 3{,}721{,}000{,}000$ times as great in light as it is in steel. Laplace estimates the velocity of gravitating action as at least 100,000,000 times the velocity of light, and the relative rigidity of speedy and complete recovery from any perturbation would, therefore, be $3.721 \times (10)^{22}$ times as great in gravitating action as in steel.

The well-known experiment of the Chladni plates shows the tendency, in uniform vibrations, to drive material particles towards the nodes and away from the internodes. According to Fourier's theorem, "every periodic vibratory motion can always, and always in one manner, be regarded as the sum of a certain number of pendulum vibrations." If two clocks, ticking nearly synchronously, are so suspended that their vibrations can be mutually communicated, they will tend to absolute synchronism. Laplace explained, on this principle, the equations of commensurability in planetary revolutions, and Kirkwood, in the paper already referred to, proposes to extend the explanation to the time of primitive nebular rupture and planetary aggregation.

All music, as is well known, consists merely of "periodic vibratory motion" in an elastic medium, and is, therefore, subject to Fourier's theorem, as well as to all other laws of gravitating or central motion which are capable of producing or affecting periodicity. The most elastic media of which we know anything, are those through which the phenomena of light and gravitation are manifested, and we might reasonably anticipate, in such media, some striking displays of musical or harmonic arrangement.

When three numbers are such that the first is to the third, as the difference between the first and second is to the difference between the second and third, they are said to be in harmonical proportion; and a series of numbers, in continued harmonical proportion, constitutes a harmonical progression. This name has been adopted because, if a musical string be divided in harmonical proportion, the different parts will vibrate in unison. It will readily be seen, upon trial, that the reciprocals of any arithmetical progression constitute a harmonical progression; *e. g.*, $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$; $\frac{2}{3}$, $\frac{2}{7}$, $\frac{2}{11}$, $\frac{2}{15}$; $\frac{c}{n-2a}$, $\frac{c}{n-a}$, $\frac{c}{n}$, $\frac{c}{n+a}$, $\frac{c}{n+2a}$; are all harmonical progressions.

The key-note of the solar system must evidently be determined by its two primary nodes—the Sun, which is about 750 times as large as all the planets, and Jupiter, which is about $2\frac{1}{2}$ times as great as the aggregate mass of all the remaining planets. In order that the other planets may vibrate in unison with their giant leader, and that there may be no discord in the continual hymn of our morning-star, it is desirable that the elastic æthereal strings, being all of similar density, should be of harmonic lengths.

Now Jupiter is so situated that it constitutes a node for each of the more remote planets, in the harmonical proportion of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$. Its own harp-string is so subdivided, as to form planetary nodes in the harmonical progression, $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc., as shown in the following table:

Theoretical.	Observed.	Theoretical.	Observed.
$6\,c$ 298·68	Nep. p[i] 297·82	$\frac{1}{7}\,c$ 7·11	Ven. 7·23
$4\,c$ 199·12	Ura. a 200·44	$\frac{1}{9}\,c$ 5·53	V. M.[ii] 5·55
$2\,c$ 99·56	Sat. a 100·00	$\frac{1}{11}\,c$ 4·53	Mer. a 4·55
c 49·78	Jup. p 49·78	$\frac{1}{13}\,c$ 3·83	Mer. 3·87
$\frac{1}{3}\,c$ 16·59	Mars a 16·44	$\frac{1}{15}\,c$ 3·32	Mer. p 3·19
$\frac{1}{5}\,c$ 9·96	Earth 10·00	$\frac{1}{17}\,c$ 2·93	Mer. $s\,p$ 2·97

[i] p, mean perihelion; a, mean aphelion; $s\,p$, secular perihelion.

[ii] Mean between mean distances of Venus and Mercury.

This is in curious analogy with Bernouilli's law of organ pipes—that if the pipe is closed at the end opposite the mouthpiece, we can, by gradually increasing the force of the air current, obtain the *uneven* harmonics of the primary tone. The key-note is sounded at Jupiter's mean perihelion, or the point of its mean maximum *vis viva*.

We may reasonably presume that the harmonic activity began even before the position of Jupiter was definitely marked out. If we start from the present mean boundary of the system, we find theoretical planetary positions at $\frac{2}{3}$, $\frac{2}{6}$, $[\frac{2}{9}]$, $\frac{2}{12}$ of Neptune's radius vector. But the immense mass of Jupiter seems to have drawn to itself the particles which might otherwise have collected at $[\frac{2}{9}]$, and to have nearly obliterated $\frac{1}{8}$ of the subsequent primitive nodes, so that the first planetary position of any importance, within Jupiter's orbit, is at $\frac{2}{36}$ instead of $\frac{2}{15}$ of the primitive radius, the present evidences of the earlier series being found at $\frac{2}{12}$, $\frac{2}{36}$, $\frac{2}{60}$, $\frac{2}{84}$ (or the equivalent $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, if Jupiter's mean perihelion is taken as the unit.) In like manner, the combined action of the principal planetary masses seems to have modified this second series within the orbit of Venus, so that Mercury's mean position is a third harmonic to Jupiter and Venus, $\frac{1}{1}$, $\frac{1}{7}$, $\frac{1}{13}$. Taking the primitive harmonics as thus modified, we form the following table:

	Mercury.	Venus.	Earth.	Mars.	Juno.	Jupiter.	Saturn.	Uranus.	Neptune.
Theoretical,	3·85	7·15	10·01	16·69	26·74	50·06	100·11	200·23	300·34
Observed,	3·87	7·23	10·00	16·44	26·71	49·78	100·00	200·44	300·34

The asteroidal position, which is here taken, is the geometrical mean of the seven obliterated Neptunian harmonics.

In the following table, the errors of the closest planetary approximations, in each of the foregoing series, are given for the purpose of comparison. In the signs, which are prefixed to the percentages of error, + indicates an excess in the theoretical, — indicates an excess in the observed, value. The "Sum" is found by dividing the difference between the sums of the theoretical and observed values, by the sums of the theoretical values.

The "Range" is the sum, without regard to sign, of the greatest positive and the greatest negative error. The greatest range (·0602) is about half of the mean planetary range from mean perihelion to mean aphelion (·1178), and less than one-third of the mean range from secular perihelion to secular aphelion (·1868). "Mean I" is $\frac{1}{8}$ of the sum of the percentages of error, taken without regard

to sign. "Mean II" is the square root of $\frac{1}{8}$ of the sum of the squares of the percentages of error. The "Average" is $\frac{1}{8}$ of the algebraic sum of the percentages.

	Bode.	Peirce.	Alexander.	Kirkwood.	Chase.
Mercury, . . .	+·0333	+·0144	+·0117	+·0144	—·0053
Venus, ...	+·0029	·0000	+·0091	+·0001	—·0115
Earth, ...	·0000	—·0071	—·0067	—·0079	+·0011
Mars, ...	—·0269	+·0180	+·0309	+·0337	+·0148
Jupiter, . . .	—·0005	—·0045	+·0059	—·0045	+·0055
Saturn, . . .	·0000	+·0152	—·0099	+·0152	+·0011
Uranus, . . .	+·0217	—·0136	+·0191	—·0136	—·0011
Neptune, . .	—·0136	·0000	·0000	·0000	·0000
Sum, ...	+·0002	—·0018	+·0054	—·0015	+·0005
Range, . . .	·0602	·0336	·0408	·0416	·0263
Mean I, . .	·0124	·0091	·0117	·0112	·0051
Mean II, . .	·0177	·0113	·0147	·0152	·0072
Average, . .	+·0021	+·0028	+·0075	+·0047	+·0006

Although the evidence, as here presented, of phyllotactic and harmonic influences is the most satisfactory, it seems impossible to doubt the efficacy of each of the tendencies upon which the other series are grounded. The superior importance, in human affairs, of the superintending and controlling Will, is almost universally acknowledged, and the most successful investigators have instinctively sought for evidences of an analogous, although infinitely higher, control, in the material universe. It should, therefore, be no cause for surprise, if we find that the most rigid mathematical tests point, from the very earliest moment of incipient nebular condensation, to such beneficent and æsthetic contrivance as, in the words of Newton, "could only proceed from the counsel and dominion of an intelligent and powerful Being." When we recognize the great truth, so tersely enunciated by Oersted, that "the laws of Nature are the thoughts of God," we may be prepared to find those thoughts expressed in manifold ways, modified by mutual interaction, but all working harmoniously together in developing the eternal designs of infinite Wisdom. As nebular condensation proceeded, through its various stages, from simple spherical aggregation to the formation of planetary nuclei, and to the ultimate gathering about those nuclei, of most of the outlying materials in our system, the laws of linear and spherical oscillatory inertia, and of cumulative cyclical action, must all have been operative. The record of their activity, as we have seen, may still be read in the measureless depths of space, where it was inscribed "in the beginning" by Him who "created the heavens and the earth."

Alsatian Report on the Centennial.—MM. Steinlen and Zuber presented to the Industrial Society of Mulhouse, a report in which they described the Philadelphia Exposition as more interesting and better organized than any that preceded it. Special thanks are given to the Franklin Institute and the American Association of Mining Engineers, for various courtesies; the calico factory of Joseph Lea and his associate, M. Eberhardt, is described as resembling a true Alsatian colony; the mechanical puddler of Messrs. W. Sellers & Co., their proposed continuous system of iron working, their establishment at Edge Moor, and their Philadelphia workshops, are all highly commended; the Corliss engine is praised for its general effect, in spite of some deficiencies of detail; the quality, finish, efficiency and moderate cost of the various steam engines, as well as of most of the machines, are extolled. Special notice is taken of the commercial iron and steel; sheet iron and steel; numerous samples of hammered work, showing the extreme density of the iron and steel, as well as the great skill of the smiths; imitation Russia iron; iron and steel rails; soft and tenacious castings, equal to the best Swedish; superior Bessemer fabrics, with particular encomiums on the rails, horse-shoes and wood-screws; hardware, including locks, padlocks, nails, saws, wrought and cast iron and enameled household utensils, edge tools, files, surgical instruments, cutlery, firearms; the general iron trade, second in importance only to that of England, a success which they attribute, in part, to the immense natural resources of the country in coal and minerals of excellent quality, partly to protective tariffs. They say: "We hold the American hardware as the most perfect in the world; free from the mannerisms as well as from the irrational and ungraceful forms which have been imposed upon us, by makers who are dealers rather than artisans."

In other departments they commend the fine displays of gold and silver ware, often displaying originality and grace, America ranking next after France, Austria and Russia, in taste and execution; superior telegraphic apparatus, and its common use in workshops, hotels, offices and private houses; excellent watches for ordinary use, the workmanship being such as soon to control the American market, and compete formidably with the European trade in other markets; furniture, of good shapes, solidly and well made, but of general bad

taste in the most costly styles; fine pianos; tissues of cotton, linen, wool and silk; very beautiful specimens of typography and engraving—the engraving of the paper-money being pronounced *splendide;* beautiful and solid bindings; crayons; metal pens of excellent workmanship; good, and often elegant, furnishings for the counting house; pretty stationery articles, rivaling the best in the world; fine writing papers of all descriptions; printing papers, as cheap as the European, in consequence of the partial substitution of straw and wood for rags; carriages, and their several parts. "The palace cars are the most beautiful and the most comfortable that we have ever seen; we cannot say as much of the greater number of those which are called first-class cars; they are certainly well built, but the passengers are cramped, badly seated, and they sometimes find themselves in too much democratic company."

They regard the general American custom of dispensing with jackets for cylinders and condensing apparatus, making no use of the escape steam except for winter heating, as ill-considered, remarking "that the Americans are not economists, in the sense of the more or less complete utilization of waste products, and the care to avoid loss; they prefer to seek new means of manufacture, or to render their present means more complete, more precise, more expeditious and more productive. A European metallurgist, with whom we traveled, and who had just been visiting some American mining works, expressed the opinion that many of the slags, from which they reaped no benefit, were at least as rich as the ores with which he was called to deal in Europe."

In Machinery Hall they notice the non-explosive boilers, hydraulic motors (especially the Geyelin turbines made by Wood); "Wood's castings, both such as are intended for turbines and the tubes for water conduits, are magnificent specimens of moulding;" shafting, hangers and pulleys; the general substitution of belting for gearing, and the great speed at which the belting is run; fine displays of blowing machines, motive powers for boats and portable fire-engines, very richly ornamented; locomotives, with steel fire boxes; machine tools, for metal and wood-working; the ingenious inventions from which "has resulted this fact, humiliating for us to acknowledge, that for most products destined for large consumption, the American fabric, in spite of labor which is paid twice or thrice what it brings elsewhere, is able to compete with the European in all markets, and

threatens to overpower it;" pumps and ventilators; forging machinery; the circular saw, with teeth of black diamonds, "working blocks of stone absolutely as if the material was wood;" spinning and weaving machinery, "carefully and solidly made;" drying machines, "of good construction;" "excellent calenders;" a new machine for stretching tissues; pantographs, for engraving printing rolls, "very well contrived and of an unexceptionable finish;" calender rolls of chilled iron, presenting admirable polish and finish, "due to a special contrivance created by the Americans, by means of which they are enabled to secure the action of hard metal cylinders with a perfection hitherto unknown;" printing presses; milling machines and apparatus; elevators; cutting, sewing, and embroidering machines; envelope machines; pin-making and setting machines; iron tubes, which "merit, in all respects, a special and most eulogistic mention," etc., etc.

In nearly all of the foregoing references, the American success is acknowledged to be quite equal to that of any other nation, and in many of them the palm of superiority is gracefully yielded. On some points, however, the criticism is unfavorable. While crediting our factories for their colossal dimensions and remarkable excellence of machinery, our lack of originality and our dependence on styles and designs stolen from France and Alsace, are sharply stated. The best educational exhibits were thought to be those from New England, but the neglect of careful attention to "little things," and particularly the frequent slovenliness of the writing, are regarded as grave defects. "Both in boys and girls the aptitudes for design, that language which speaks to the eyes, seem ordinary to the last degree. The needlework is careless and devoid of taste." Though the American belting is the best in the world, the tanning is imperfect, "The United States continue dependent upon France for skins and leather suited for the best shoemaking; the finest bootings are also of French production, and there is no reason to fear that it will soon be otherwise. We should state, in this connection, that the same thing is true for all objects of *grand luxe*—dresses, gloves, fine perfumery, toilet articles, furniture, bronzes, silver and plated ware, jewelry, mirrors, glassware, porcelain, cloths, tissues, carpets, paperhangings, morocco work, and other goods designed for a select patronage."

After noticing the displays in Agricultural Hall, and commenting on the principal elements of production, such as raw materials, motive power, manual labor, prices current, rates of interest, and means of communication, the conclusion is drawn that our wonderful development is not factitious, and that we are perfectly fitted to compete with Europe. "Moreover, let us not forget that its commerce and its industry are governed by a people of devouring activity, eager for gain, even foolishly enterprising, and convinced to the last degree of its superiority over all other nations. This confidence, which has of late years lost no occasion to assert itself, has been again exalted by the spectacle which the Philadelphia Exposition offered to the Americans. They flocked thither in crowds, from all corners of the Republic, and returned to their homes prouder than ever of their title of Yankees. . . . The Americans owe this sentiment in great part to their professional education. Most of them have passed through a serious manual apprenticeship, and have learned to be good workmen before becoming masters. Coming thus into early contact with the realities and the details of their handicraft, they gain a surer eye, a better developed practical sense, and at the same time a greater confidence in themselves.

"Shadows are not wanting, however, in the tableau of the future industrial grandeur of the United States. The spirit of enterprise has its dangers, and may lead to formidable crises: the political situation of the country, more unsettled than elsewhere, will often injure the development of business; in fine, the want of good industrial traditions, the fruits of a long experience, and of extended commercial relations, will shackle, in some measure, the progress of American industry. But while these different causes may affect its prosperity, and diminish its benefits, they do not lessen the danger which results for us from the coming into line of a formidable competitor. . . . We not only bring you the firm conviction that the European importations into the United States are destined in a very near future, to embrace only articles of special manufacture, novelties of all kinds, and objects of luxury and taste; we are equally persuaded that, before long, the articles of large consumption, made in North America, will compete with ours in all foreign markets."—*Bulletin de la Soc. Indust.*, March–May, 1877. C.

AN ELEMENTARY DISCUSSION OF THE PRINCIPLE OF LEAST SQUARES.

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The Method of Least Squares is in constant use among engineers, for the adjustment of observations, but a simple and yet perfectly satisfactory proof of its main principle is still a desideratum. In this article, I wish to bring to notice a demonstration which has not yet found a place in English books, and which is far more simple and conclusive than the one usually given. I shall attempt, in writing it, to everywhere simplify and illustrate the reasoning of the proof, and to give enough of the fundamental principles of Probability to render the whole argument plain to those entirely unacquainted with the subject.

The proof alluded to is that of Dr. Hagen, who now, at a ripe old age, enjoys the honor of being one of the greatest hydraulic engineers of the century, and was first published at Berlin, in 1837, in his book entitled "Grundzüge der Wahrscheinlichkeitsrechnung." Although often used in subsequent German and French works, it received no attention from English mathematicians until 1865, when Prof. Tait re-discovered it in a greatly modified and much less satisfactory form, and published it in Vol. xxiv of the *Edinburgh Transactions*. This, and a paper by Mr. Kummell, of the U. S. Lake Survey, in *The Analyst*, 1876, Vol. iii, p. 133, are the only sources of information concerning it in the English language.[1] Mr. Kummell's discussion, although very abbreviated, and requiring in its readers a previous knowledge of the subject, was very welcome to mathematicians, and it contains one or two modifications of the German method of presentation, which considerably shorten the algebraic work.

Hagen's proof has, I think, but one difficulty, and that lies in the fundamental axiom or hypothesis upon which it is based. This difficulty is very slight compared with those in Gauss's or Laplace's proofs, while the mathematical work is vastly simpler. For the be-

[1] See also, Price's *Infinitesimal Calculus* (London, 1865), Vol. ii, pp. 376–379.

ginner, it seems to me that Hagen's proof is decidedly the best of the thirteen[1] which have been offered.

I first proceed to give a few fundamental definitions relating to the subject:

DEFINITIONS.

When several observations, or sets of observations, are made to determine the magnitude of a quantity (for example, the length of a line), the results do not agree. All of these results cannot be correct, and each one of them can be regarded only as an approximation to the truth. The absolutely *true value* of the quantity in question we can never obtain, or, at least, be never sure that we have obtained; and, instead of it, we must accept and use a value derived from the combination of our observations which shall be the *most probable value.* The most probable value is that in which we have the greatest degree of confidence as being near to the true value.

The Method of Least Squares furnishes rules and processes for the determination of the most probable values of observed quantities. The reason of the name "Least Squares," will be plain in the sequel.

An error is the difference between the true value of a quantity and the value found by observation. If the true value exceeds an observed value, the error is called a positive one; but if it is less than an observed value, the error is called negative.

Constant errors are those which always, under the same circumstances, have the same values, and which, therefore, strictly speaking, are not errors, but the results of law. Such are: the effect of refraction upon a vertical angle, the effects of heat in changing the length of a measuring rod, the effect of incorrect graduation, the result of using an instrument not properly adjusted, etc. All such errors may be removed from the observations by a proper method of using the instrument, or by applying computed corrections to the measured results, and they *must* be so removed before the Method of Least Squares is used.

Accidental errors are those that remain after all constant ones have been eliminated from the observations. Such, for example, are the errors in leveling—arising from sudden expansions and contractions of the instrument, or from the effects of the wind, or those often observed in sighting across a river—arising from the anomalous and changing refraction of the atmosphere. More than all, however,

[1] For brief notes on these thirteen proofs, see *The Analyst*, for March, 1877.

they arise from imperfections of our touch and sight, which render it impossible for us to handle our instruments with perfect delicacy, or estimate accurately small divisions of their graduation. These are the errors which the method of least squares discusses and adjusts. Being produced by many unknown causes, whose laws of action are also unknown, they can be discussed only by the laws of chance or probability. We must, then, as preliminary to our investigation, state and exemplify the first elements of the mathematical theory of Probability.

PRINCIPLES OF PROBABILITY.

The word *probability*, in mathematics, denotes a fraction between unity and zero, which indicates the degree of our mental confidence in the occurrence of an event. If the event is *certain* to happen, its probability is 1, if *impossible*, its probability is 0, and between these limits there may be every degree of belief and of probability. Thus, the probability that the sun will rise to-morrow morning, is about $\frac{2129001}{2190002}$, or almost a certainty; the probability of not drawing a prize, in a lottery of 250 tickets, 25 of which are prizes and the rest blanks, is $\frac{9}{10}$; the probability that head will be thrown in tossing a coin, is $\frac{1}{2}$; the probability of throwing an ace with a single die, in one trial, is $\frac{1}{6}$; and the probability that the dealer in a game of whist will have all the trumps, is $\frac{1}{158753389900}$, or almost an impossibility.

The probability of the happening of an event is a fraction whose denominator denotes the whole number of possible cases, and whose numerator denotes the number of cases in which the event may happen, all cases being supposed equally probable. Thus, in throwing a die, there are six possible cases, in five of which an ace will not be thrown, so the probability of not throwing an ace is $\frac{5}{6}$; if a pack of 52 cards contains 4 kings, and one card be drawn, the probability that it will be a king is $\frac{4}{52} = \frac{1}{13}$; if a bag contains 23 white and 2 black balls, the probability of drawing a white ball is $\frac{23}{25}$, and that of drawing a black ball is $\frac{2}{25}$.

The probability of the happening of an event, plus the probability of its failing, is unity. For it is *certain* that the event must either happen or fail. Thus, the probability of throwing an ace with a single die, in one trial, is $\frac{1}{6}$, the probability of not throwing an ace is $\frac{5}{6}$, and $\frac{1}{6} + \frac{5}{6} = 1$.

If an event may happen in different independent ways, the probability of its happening is the sum of the separate simple probabilities.

This follows immediately from the above definition of probability, and may be illustrated by the following examples: the probability of throwing either an ace or a deuce with a die, in one trial, is $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$; the probability of throwing either a head or a tail, in tossing a coin, is $\frac{1}{2}+\frac{1}{2}=1$; the probability of drawing a ball which is not white, from a bag containing 5 black, 3 red and 7 white balls, is $\frac{5}{15}+\frac{3}{15}=\frac{8}{15}$.

The probability of the happening of several independent events, is equal to the product of their separate simple probabilities. To prove this, let there be two bags, one containing 7 black and 9 white balls, and the other containing 4 black and 11 white balls, and, let us ask, what is the probability in drawing from both bags at once—that both balls drawn will be black? Since each ball in the first bag may form a pair with each one in the second, there are 16×15 possible ways of drawing two balls, and since each of the 7 black balls may form a pair with each of the 4 black balls, there are 7×4 cases favorable to the drawing of 2 black balls. Hence, by the definition above, the probability of drawing 2 black balls, is $\frac{7\times4}{16\times15}=\frac{7}{60}$. But the simple probability of drawing a black ball from the first bag, is $\frac{7}{16}$; that of drawing one from the second, is $\frac{4}{15}$, and $\frac{7}{16}\times\frac{4}{15}=\frac{7}{60}$. The rule is, therefore, proved.

As an illustration of the above principles, let us consider the following question: A solves three out of every four problems proposed, and B solves one out of every three; a certain problem being proposed, what is the probability that it will be solved if both try. By the definition, the simple probability that A will solve it, is $\frac{3}{4}$; that B will solve it, is $\frac{1}{3}$; that A will fail to solve it, is $\frac{1}{4}$; and that B will fail to solve it, is $\frac{2}{3}$. Now the problem may be solved in one of three independent ways, viz.:

1. When A solves and B fails.
2. When A solves and B solves.
3. When A fails and B solves.

Each of these ways is composed of two independent events, and the probability of each is found by the multiplication of the separate simple probabilities; or,

Probability of $1=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$.
Probability of $2=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$.
Probability of $3=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$.

The sum of these, or $\frac{1}{2}+\frac{1}{4}+\frac{1}{12}=\frac{5}{6}$, is the probability sought. The same result may be found in another way, thus: the problem will not be solved if both fail; the probability of this is $\frac{1}{2}\times\frac{2}{3}=\frac{1}{6}$; but it is certain that they will either succeed or fail, hence $1-\frac{1}{6}=\frac{5}{6}$, is the probability of succeeding.

The most probable event is that which has the greatest probability among all the probabilities of all the possible events. Thus, if 10 coins are thrown, the most probable case is that 5 of them will be heads and 5 tails; if 2 dice are thrown, the probability that the sum of the points will be 7, is greater than that for any other number; and, as we shall show in the sequel, the average of several direct measurements of equal precision, has a greater probability of being the true value than any other result that can be derived from those observations.

To show how this greatest probability (and hence, the most probable event) may be determined for simple cases, let us consider the following problem: Six dice are thrown at once; which is the most probable of the following cases?

1. Six aces turn up.
2. But five aces turn up.
3. But four aces turn up.
4. But three aces turn up.
5. But two aces turn up.
6. But one ace turns up.
7. No aces turn up.

Let p be the probability of throwing an ace with a single die in one trial, and q the probability of not throwing it. If 6 dice be thrown, the probability that all will be aces is that of the concurrence of 6 independent events, each of the simple probability p, or

$$p\,.\,p\,.\,p\,.\,p\,.\,p\,.\,p=p^6.$$

The probability that *any assigned die* will not turn up ace, and that the other 5 will be aces, is $q\,.\,p\,.\,p\,.\,p\,.\,p\,.\,p=p^5\,q$; and, since there are 6 dice, this may happen in 6 ways, and hence,

$$6\,p^5\,q$$

is the probability that only 5 will turn up aces. Similarly, the probability that *two assigned dice* will not be aces, and that the rest

will be aces, is $q \cdot q \cdot p \cdot p \cdot p \cdot p = p^4 q^2$; and, since this may happen in $\frac{1}{2} \cdot 6 \cdot 5 = 15$ ways,[1] we have

$$15\, p^4 q^2$$

as the probability that only 4 aces will turn up. We see, then, that the successive terms of the binomial expansion,

$$(p+q)^6 = p^6 + 6\, p^5 q + 15\, p^4 q^2 + 20\, p^3 q^3 + 15\, p^2 q^4 + 6\, p\, q^5 + q^6,$$

represent the respective probabilities of the seven cases. Placing for p its value $\frac{1}{6}$, and for q its value $\frac{5}{6}$, we find:

1. Probability of six aces $= p^6 = \frac{1}{46656}$.
2. Probability of but five aces $= 6\, p^5 q = \frac{30}{46656}$.
3. Probability of but four aces $= 15\, p^4 q^2 = \frac{375}{46656}$.
4. Probability of but three aces $= 20\, p^3 q^3 = \frac{2500}{46656}$.
5. Probability of but two aces $= 15\, p^2 q^4 = \frac{9375}{46656}$.
6. Probability of but one ace $= 6\, p\, q^5 = \frac{18750}{46656}$.
7. Probability of no aces $= q^6 = \frac{15625}{46656}$.

From which we see that Case 6 is the most probable. The sum of the seven probabilities is unity, since one of the cases is certain to occur.

So in general, if p be the probability of the happening of an event, and q the probability of its failing, so that $p+q=1$, *and if there be m such events, the terms of the binomial expansion,*

$$(p+q)^m = p^m + m\, p^{m-1} q + \frac{m\,(m-1)}{1 \cdot 2} p^{m-2} q^2 + \frac{m\,(m-1)\,(m-2)}{1 \cdot 2 \cdot 3.} p^{m-3} q^3 + \ldots$$

give the probabilities of the several cases, the first term being the probability that all the events will happen, the second the probability that $m-1$ will happen and 1 fail, the third the probability that $m-2$ will happen and 2 fail, and the $n+1$ term the probability that $m-n$ will happen and n fail. For example: If 5 coins are thrown, find the probability that all will be heads, that 4 will be heads and 1 a tail, etc. Here p, or the probability of throwing a head with a single coin in 1 trial, is $\frac{1}{2}$; q is also $\frac{1}{2}$, and m is equal to 5. Substituting these in the binomial formula, we find

$$\frac{1}{32}, \frac{5}{32}, \frac{10}{32}, \frac{10}{32}, \frac{5}{32} \text{ and } \frac{1}{32},$$

as the respective probabilities of the six possible cases.

[1] See the Theory of Combinations and Permutations in any algebra.

THE PROBABILITY OF ERROR.

If a person accustomed to the use of the rifle shoots 500 times at a target, all the bullets will not hit the central bull's-eye, and some, perhaps, will not even hit the target. The deviation of each bullet from the centre of the target is an *error*, and, furthermore, an accidental error, produced by changes in the wind, imperfections in the sight of the marksman, etc.; for all constant errors, such as the effect of gravitation, are assumed to be eliminated in the sighting of the rifle. An examination of the bullet marks on the target shows us, however, that these accidental errors are arranged around the central point in a very regular and symmetrical manner. First, we observe that small errors, or deviations from the centre, are more frequent than large ones; secondly, that they are arranged symmetrically around the centre, so that at equal distances above, below and on each side of that centre, the number of marks in a square inch is approximately the same; and thirdly, we recognize the fact that very large errors (such, for instance, as a deviation of half a mile from the centre) do not occur. Further, the greater the skill of the marksman, the more closely will his bullets be grouped around the centre.

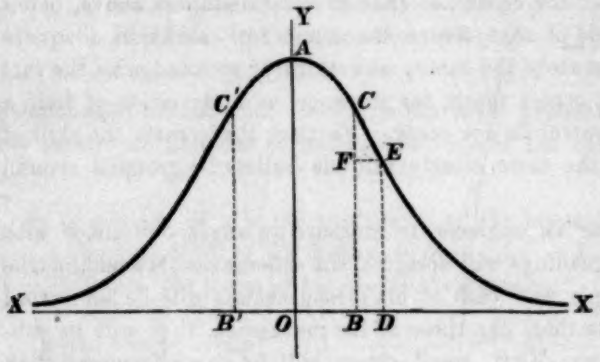
Again, suppose an engineer to measure an angle 500 times with equal care, the readings will disagree, the differences between the true value of the angle and each of his measurements will be an error; and we recognize that, like those of the marksmen, they will be subject to three laws: First, small errors will be more frequent than large ones; second, errors in excess and deficiency will be equally numerous; and third, very large errors, like those of 2°, do not occur.

In any set of carefully made observations, then, the probability of a small error is greater than that of a large one, the probability of a positive error is the same as that of an equal negative error, and the probability of a very large error is zero. Thus the probability of an error is a *function* of the error, so that if x represents any error, and y its probability, we may write

$$y = f(x),$$

an equation stating that y is a function of x, that is, that y is in some way related to x and dependent upon x for its value.

If we regard x as an abscissa, and y as the corresponding ordinate, the equation $y = f(x)$ represents a curve, which must be of such form as to agree with the three laws adopted above, viz., its maximum ordinate $O\,A$ must correspond to the error zero; it must be symmetrical with respect to the axis of Y, so that if $O\,B = O\,B'$, $B\,C$ will equal $B'\,C'$; as x increases numerically, y must rapidly decrease, and when x becomes very large, y must be very small. The figure represents such a curve, $O\,B$ and $O\,D$ being two positive errors, having the respective probabilities $B\,C$ and $D\,E$, and $O\,B'$ being a negative error whose probability is $B'\,C'$. The particular dimensions of the curve will depend upon the precision of the observations, and each set of measurements will have a distinct curve of its own, whose general form, however, must be as shown in the figure.



If now we can find the value of $\varphi(x)$ so that the probability y of any error x is given directly in terms of x, we shall have the equation of the curve which represents the law of probability of error. This equation we now proceed to deduce.

HAGEN'S DEDUCTION OF THE LAW OF ERROR.

The demonstration of Hagen is based upon the following axiom or hypothesis: *An accidental error of observation results from the combination of a very large number of small elementary errors, which are all equal, and each of which is equally likely to be positive or negative.* Thus, suppose that by several observations with a leveling instrument the difference of level between two points has been determined. This value is greater or less than the *true* difference of level by a small error x. This error x is the result of numerous causes, acting at

every observation; the instrument is not perfectly level, the wind shakes it, the sun's heat expands one side of it, the level's bubbles are not accurately made, the spider lines are not in perfect adjustment, the focus is not exactly set, the glass gives an indistinct definition, the eye of the observer is not in perfect order, there is irregular refraction due to the atmosphere, the man at the rod does not hold it vertical, the graduation of the rod is poor, the target is not properly clamped, the rod-man errs in taking the reading, etc. These causes may be subdivided into others, each of which produces a small elementary error Δx, and from our ignorance of the laws producing them we must consider that each Δx tends to make the observed result too large as often as it does too small, that is, each is equally likely to be positive or negative. The number of these elementary errors Δx is very great, and we designate it by m. If $\frac{1}{2}\,m$ are positive and $\frac{1}{2}\,m$ negative, the resulting error x will be 0; if more than $\frac{1}{2}\,m$ are $+$, the resulting error x will be positive.

Now it is evident that it is more probable that the number of positive elementary errors should be approximately equal to the number of negative ones than that either should be markedly in excess, and that the probability of the elementary errors being either all $+$ or all $-$ is exceedingly small. In the first case the actual error is small, and in the second large, and so the probabilities of small errors are the greatest, and the probability of a large error is practically zero. These correspond to the properties which we have shown the probability curve must possess.

Since by the axiom any elementary error Δx is equally likely to be positive or negative, the probability that Δx will be positive is $\frac{1}{2}$, and that it will be negative is also $\frac{1}{2}$. The probability that all of the m elementary errors will be positive, is hence $(\frac{1}{2})^m$, the probability that $m-1$ will be positive and 1 negative is $m\,(\frac{1}{2})^{m-1}(\frac{1}{2})^1$, and the probabilities of all the respective cases will be given by the corresponding terms of the binomial formula. When all of the m elementary errors are positive, the resulting error of observation is $+\,m\,.\,\Delta x$, when $m-1$ are positive and 1 negative, the resulting error is $+\,(m-1)\,\Delta x - \Delta x$, or $+\,(m-2)\,\Delta x$. If $m-n$ elementary errors are $+$ and the remaining n are $-$, the resulting error is $+\,(m-n)\,\Delta x - n\,.\,\Delta x$, or $+\,(m-2n)\,\Delta x$, and the probability of this particular combination is given by the $m+1$th term of the

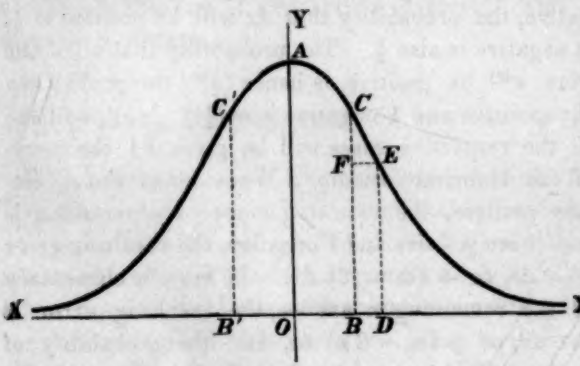
binomial expansion, p and q being each equal to $\frac{1}{2}$. We thus readily deduce the following table:

Elementary Errors Δx.	Resulting Error x.	Its Probability y.
If m are + and 0 are −	$m\,\Delta x$	$(\frac{1}{2})^m$
If $m-1$ are + and 1 is −	$(m-2)\,\Delta x$	$m(\frac{1}{2})^m$
If $m-2$ are + and 2 are −	$(m-4)\,\Delta x$	$\frac{m\,(m-1)}{1.2}(\frac{1}{2})^m$
If $m-3$ are + and 3 are −	$(m-6)\,\Delta x$	$\frac{m\,(m-1)\,(m-2)}{1.2.3}(\frac{1}{2})^m$
.....	.....	....
If $m-n$ are + and n are −	$(m-2n)\,\Delta x$	$\frac{m\,(m-1)\,(m-2)\,\ldots\ldots\,(m-n+1)}{1.2.3\,\ldots\ldots\,n}(\frac{1}{2})^m$
If $m-n-1$ are + and $n+1$ are −	$(m-2n-2)\,\Delta x$	$\frac{m\,(m-1)\,(m-2)\,\ldots\ldots\,(m-n)}{1.2.3\,\ldots\ldots\,n+1}(\frac{1}{2})^m$
....	... .	.....

In the curve $y=\varphi(x)$ let OD be any error x, DE its probability y, also let $OB=x'$ be an error less in magnitude, and BC its corresponding probability y'. Then we see from the figure that

$$\text{limit}\,\frac{CF}{FE}=\text{limit}\,\frac{y-y'}{x-x'}=\frac{dy}{dx},$$

is the differential equation of the curve. To determine from this the equation giving the law of probability of error, we have simply to find $\frac{y-y'}{x-x'}$ in terms of y and x, pass to the limit, place it equal to the first differential coefficient $\frac{dy}{dx}$, and perform the integration.



Since x and x' represent any errors, x' being the less, we may take

$$x = (m - 2n)\,\Delta x \text{ and } x' = (m - 2n - 2)\,\Delta x,$$

then, by subtraction, we have

$$x - x' = 2\,\Delta x.$$

The probabilities corresponding to x and x', we have found to be

$$y = \frac{m(m-1)\ldots\ldots(m-n+1)}{1.2.3\ldots\ldots n}\left(\tfrac{1}{2}\right)^m,$$

$$y' = \frac{m(m-1)\ldots\ldots(m-n)}{1.2.3\ldots\ldots n+1}\left(\tfrac{1}{2}\right)^m.$$

From these we see that

$$y' = y\,\frac{m-n}{n+1}.$$

Subtracting both sides of this equation from y, gives

$$y - y' = y - y\,\frac{m-n}{n+1} = y\,\frac{2n-m+1}{n+1},$$

and inserting in this the value of n, taken from the equation $x = (m - 2n)\,\Delta x$, it becomes

$$y - y' = y\,\frac{2\,\Delta x - 2x}{(m+2)\,\Delta x - x}.$$

Dividing this by the equation $x - x' = 2\,\Delta x$, member by member, we have

$$\frac{y-y'}{x-x'} = y\,\frac{2\,\Delta x - 2x}{(m-2)\,2\,\Delta x^2 - 2x\,\Delta x}.$$

When the elementary errors Δx are indefinitely small, the curve becomes continuous, and the limit of $\frac{y-y'}{x-x'}$ is reached; Δx then vanishes, in comparison with x, and

$$\text{limit}\,\frac{y-y'}{x-x'} = -\,\frac{2yx}{2\,(m\ 2) + \Delta x^2}.$$

The quantity $(m + 2)\,\Delta x^2$ requires particular attention; $m + 2$ is an indefinitely great number, and Δx^2 is an indefinitely small quantity of the same kind as x; the product $(m + 2)\,\Delta x^2$ is hence a positive *constant*, which, for the present, we have no means of determining. Placing, then,

$$\frac{1}{2\,(m+2)\,\Delta x^2} = \text{a positive constant} = h^2,$$

we have,

$$\text{limit}\,\frac{y-y'}{x-x'} = -\,2h^2\,y\,x;$$

the differential equation is, hence,

$$\frac{dy}{dx} = -2h^2 y x,$$

or, $$\frac{dy}{y} = -2h^2 x\, dx.$$

Integrating this, we have

$$\log y = -h^2 x^2 + k,$$

in which k is the constant of integration, and the logarithm is taken in the Napierian system. Passing from logarithms to numbers, we obtain

$$y = e^{-h^2x^2 + k} = e^{-h^2x^2} e^{k},$$

in which e is the base of the Napierian system (the number 2·71828). Since e^k = a constant, this may be written

$$y = c\, e^{-h^2x^2},$$

and this is the equation of the probability curve, or the equation giving the law of probability of error.

DISCUSSION OF THE EQUATION.

Let us test this equation, and see whether it conforms to the conditions which we imposed upon the law $y = \varphi(x)$ at the beginning of our investigation.

In the equation,

$$y = c\, e^{-h^2x^2} = \frac{c}{e^{h^2x^2}},$$

y is the probability of the error x, and c and h are constants. Since the square of x only enters, equal positive and negative values of x will give equal values of y. Hence the curve is symmetrical with respect to the axis of Y, and this corresponds with the requirement that equal positive and negative errors should be equally probable. The greatest value of y will obtain when $e^{h^2x^2}$ is the smallest possible; this occurs when $x = 0$; y is then equal to c, and for all other values of x it is a fractional part of c; this agrees with the requirement that small errors should be more probable than large ones. As x increases numerically, $e^{h^2x^2}$ increases very rapidly, and when x is large, y is exceedingly small; and this agrees with the axiom that very large errors seldom occur. By computing the values of y for corresponding values of x, regarding c and h as unity, we may gain

a clear idea of the form of the curve. The following are a few such values:

For $x = 0{\cdot}0$	$y = 1{\cdot}$
$0{\cdot}2$	$0{\cdot}9608$
$0{\cdot}4$	$0{\cdot}8521$
$0{\cdot}6$	$0{\cdot}6977$
$0{\cdot}8$	$0{\cdot}5273$
$1{\cdot}0$	$0{\cdot}3679$
$1{\cdot}2$	$0{\cdot}2370$
$1{\cdot}4$	$0{\cdot}1409$
$1{\cdot}6$	$0{\cdot}0773$
$1{\cdot}8$	$0{\cdot}0392$
$2{\cdot}0$	$0{\cdot}0183$
$3\,0$	$0{\cdot}0001$
∞	$0{\cdot}$

From these values the above figure has been plotted, the vertical scale being double the horizontal. The curve crosses the axis of Y at right angles, and has the axis of X as asymptotes; the inflection point is at E, corresponding to $x = 0{\cdot}707$.

(It can be shown that the hypothesis of continuous errors adopted above, requires that c must be an extremely small fraction; the probability of the occurrence of any particular error being a fractional part of c, is, hence, exceedingly small. But the sum of all the possible values of y is equal to unity.)

The probability y' of any particular error x' will depend upon the constants c and h, and will be the larger the greater those constants are; c and h are hence related to the *precision* of the observations. The more carefully and precisely measurements are executed, the larger will be the values of c and h.

THE PRINCIPLE OF LEAST SQUARES.

Suppose, now, a number of observations be made to determine the values of certain quantities. However carefully the measurements be made, there will be disagreement in the results, and hence we can never be sure that any adjustment, which we may make, will give us the absolutely true values of the quantities. The most we can do, is to determine approximate results, which shall be the *most probable* values, and moreover be rendered most probable by the existence of the observations themselves.

The preceding principles of the probability of error afford a general rule for the determination of the most probable values of observed quantities. To deduce it in its simplest form, let us suppose that the observations are equally good, or made with the same degree of precision, then the constants c and h, which measure that precision, will be the same for all the observations. Let the errors of observation, which are the differences between the measurements and the corresponding *true* values, be denoted by x_1, x_2, x_3, etc.

Now, if x be any error, and y its probability, the law of error deduced above, gives the relation

$$y = c\,e^{-h^2x^2}.$$

Hence, for the errors actually committed, we have

Probability of the error	$x_1 = y_1 = c\,e^{-h^2x_1^2}$	
" "	$x_2 = y_2 = c\,e^{-h^2x_2^2}$	
" "	$x_3 = y_3 = c\,e^{-h^2x_3^2}$	
etc.	etc.	etc.

Now from the above principles the probability of committing the particular system of independent errors, x_1, x_2, x_3, etc., is the product of their respective simple probabilities, or

$$y_1 \cdot y_2 \cdot y_3 \cdot \text{etc.} = c\,e^{-h^2x_1^2} \cdot c\,e^{-h^2x_2^2} \cdot c\,e^{-h^2x_3^2} \cdot \text{etc.}$$

Designating this product by P, and reducing, we have

$$P = c\,c\,c \ldots\ldots e^{-h^2(x_1^2 + x_2^2 + x_3^2 + \ldots\ldots)}$$

as the expression for the probability of committing a particular system of errors.

What the true values of the measured quantities, or of the actual errors, are, we cannot hope to determine, and we must be content with finding their most probable values. Now the most probable system of errors is that which has the greatest probability among all the probabilities of all the possible systems. The most probable system, then, is that for which the product P is a *maximum*, and P in the above expression will be a maximum when the exponent of e is a maximum, that is, when

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + \text{etc.} = \text{a } \textit{minimum}.$$

The errors found by this condition are called *residual* errors, to distinguish them from true errors. As these errors are functions of the observed quantities, the most probable system of errors must corre-

spond to the most probable system of values for the latter, and we have then the principle: *The most probable values of quantities, measured several times with equal care, are those which render the sum of the squares of the residual errors a* MINIMUM. Hence the term "Least Squares."

For example, suppose that a *single* quantity, whose true value is z, is directly measured n times with equal care, and let the measurements give the values $M_1, M_2, M_3, \ldots\ldots M_n$. Then the errors committed are

$$z - M_1 = x_1, \quad z - M_2 = x_2, \quad z - M_3 = x_3, \ldots\ldots z - M_n = x_n,$$

and the sum of their squares is

$$(z - M_1)^2 + (z - M_2)^2 + (z - M_3)^2 + \ldots\ldots + (z - M_n)^2,$$

and by the above principle this is to be made a minimum to give the most probable value of z. Applying the usual method for determining minima, we differentiate the expression thus,

$$2(z - M_1)\,dz + 2(z - M_2)\,dz + 2(z - M_3)\,dz + \ldots + 2(z - M_n)\,dz,$$

place it equal to zero, and divide by $2\,dz$, giving

$$(z - M_1) + (z - M_2) + (z - M_3) + \ldots\ldots + (z - M_n) = 0,$$

in which z represents the most probable, and not the true, value. Solving this equation we find

$$z = \frac{M_1 + M_2 + M_3 + \ldots\ldots + M_n}{n},$$

or, the most probable value of a quantity observed directly n times with equal care, is found by taking the arithmetical mean of the measurements.

If the observations are not of equal precision, the constants c and h are different for each measurement. For the observation M_1 we have c_1 and h_1, and the error x_1, for M_2 we have c_2 and h_2, and the error x_2, and so on. The product P here becomes

$$P = c_1 c_2 c_3 \ldots\ldots e^{-(h_1^2 x_1^2 + h_2^2 x_2^2 + h_3^2 x_3^2 + \ldots\ldots)},$$

and the most probable value is when it is a maximum, that is, when

$$h_1^2 x_1^2 + h_2^2 x_2^2 + h_3^2 x_3^2 + \ldots\ldots = \text{a } \textit{minimum}.$$

The development of this principle to the deduction of rules for the adjustment of observations, together with the theory of the comparison of observations by means of their probable errors, constitutes the Method of Least Squares. About a hundred books have been written on this subject since 1805, the date of Legendre's discovery of the principle, but only three or four of them are in the English language.

STEAM BOILERS AND ENGINES FOR HIGH PRESSURES.[1]

By Mr. Loftus Perkins, of London.

The object of this paper is to bring before the Institution plans for generating high-pressure steam, say from 250 lbs. to 1000 lbs. per square inch, and working it with great expansion and perfect safety, in conjunction with simplicity and durability. Sixteen years ago the author, conjointly with Professor Williamson, read a paper on this subject at a meeting of this Institution in 1861. The engine and boiler then described have been in use ever since, and recently became the property of a gentleman who for several years has had another boiler and engine on the same system at work. The boiler and engine of 1861 are to be re-erected at the new works of the Sub-Wealden Gypsum Company, at Battle near Hastings, and are to form part of a steam plant consisting of three sets of boilers and engines, etc., on this system. Since 1861 many improvements have been effected, and are embodied in the engines recently constructed and illustrated in the accompanying diagrams.

In generating steam of the high pressure required to realize a fuller benefit of expansion, it has previously been found impossible to combine in the boiler great strength and safety with durability; if the former are secured, by reducing the internal dimensions and capacity of the boiler, the impurities passed in are fatal to the latter. In working a marine engine which was designed to use water distilled from sea-water, the author found that, although extreme care was taken to separate all the impurities from it before it was introduced into the boiler, the internal surfaces were in the course of time seriously injured. In the same manner, ordinary marine boilers using surface condensation have been injured when there has been an insufficient supply of sea-water to form a protecting scale on the exposed internal surfaces. This led the author to seek for a remedy, which he succeeded in discovering, and adopted with absolute success. This was the use of nothing but fresh water, or distilled fresh water in the boiler, used over and over again, without any admixture of sea-water or the products of sea-water, and this was easily accomplished, as the

[1] Paper read before the Institution of Mechanical Engineers.

machinery in question had been designed to avoid any leak whatever, and the amount of waste that did take place from glands, etc., was so small in quantity, that no practical inconvenience was found in providing the small supply of fresh water required to make good the waste that did occur.

The means taken to secure the soundness of all the joints and parts of the machinery were the same as those which had previously proved successful in the manufacture of the high-pressure heating apparatus which the author and his firm have been making for upwards of 45 years, and which has continued to work with the same water with which it was originally charged, without any destructive effect on the internal surfaces. Many sets of this heating apparatus have been working 40 years without decay; and some specimens of tubes from the boiler that was described in the former paper in 1861, which were cut out of this boiler for the Admiralty Boiler Committee in 1874, were found to be in such a remarkably good state of preservation that the committee made a special report on the system, which was laid before Parliament, and the specimens referred to are now shown on the table by the kind permission of the committee. The committee examined the condition of the boiler and cylinders of the engine at the writer's works, which were opened for the purpose, in the presence of the committee; and found the tubes of the boiler in a remarkably good state of preservation after having been in use nearly thirteen years, and the piston packing and valve rings made of the special metal were found in excellent condition after eighteen months' working without lubrication since last examined.

The possibility of using water which did not injure the internal surface of the boiler enabled the author to design the boiler on a system that combines maximum strength and safety. The horizontal tubes are $2\frac{1}{4}$ in. internal and 3 in. external diameter, excepting the steam collecting tube, which is 4 in. internal and $5\frac{1}{2}$ in. external diameter. The horizontal tubes are welded up at each end $\frac{1}{2}$ in. thick, and connected by small vertical tubes of $\frac{1}{8}$ in. internal and $1\frac{5}{16}$ in. external diameter. The firebox is formed of tubes bent into a rectangular shape placed $1\frac{1}{4}$ in. apart, and connected by numerous small vertical tubes $\frac{1}{8}$ in. internal diameter. The body of the boiler is made of a number of vertical sections, composed each of eleven tubes, connected at each end by a vertical one; these sections are connected at both

ends by a vertical tube to the top ring of the firebox, and by another to the steam collecting tube. The whole of the boiler is surrounded by a double casing of thin sheet iron, filled up with vegetable black to avoid loss of heat. Every tube is separately proved by hydraulic pressure to 4000 lbs. per square inch, and the boiler in its complete state to 2000 lbs., this pressure remaining in some hours without showing any signs of leakage. Experience of a very extensive character has proved that this construction of boiler can be worked safely, with great regularity, and without priming, and that the steam produced is remarkable for its freedom from moisture. The area through the vertical connecting tubes is found ample for allowing of the free escape of the steam, and for the prevention of injury from overheating of the tubes in contact with the flame. Injury arising from a prolonged stoppage of the feed supply is a casualty to which all boilers are liable, but with this construction of boiler the small capacity of the sections reduces to a minimum any danger arising from such injury, and facilitates rapidity of repair.

The engine has three cylinders: the first is a single-acting high-pressure cylinder, and the second also a single-acting cylinder, four times the capacity of the first; these two cylinders are bolted together in the same straight line, and have a common piston-rod. The third cylinder is double-acting, four times the capacity of the second, and its piston-rod is connected to a crank at right angles to the other crank.

Having safely generated steam of high pressure at say 350 lbs. per square inch, a serious difficulty has to be overcome in using it, from the high temperature affecting the lubrication of the pistons and packing of the glands. This difficulty the author has succeeded in overcoming by introducing the high-pressure steam into the upper end of the first cylinder, where there is no gland, and where the piston is formed so as to require no lubricating material. The steam is cut off at about half stroke in this cylinder, and when it is admitted for the return stroke into the bottom of the second cylinder, of four times the area, the temperature is so much reduced as to cause no difficulty when brought into contact with the piston-rod gland. From the bottom of the second cylinder the steam expands into the top of the same cylinder, which is of larger capacity than the bottom, and serves as a chamber, and is in direct communication with the valve box of the third cylinder; this last is double-acting, and is arranged to cut

off at about a quarter stroke, and at the termination of the stroke exhausts into the condenser, with a total expansion of about thirty-two times. All the cylinders are jacketed with wrought-iron tubes, which are cast in the metal, and supplied with steam direct from the boiler, the condensed water from the jackets being conveyed to the hot well. The whole of the cylinders and valve boxes, etc., are enclosed with a double case of thin sheet iron, filled in with vegetable black to prevent the escape of heat, and at the same time to maintain all the parts at a high temperature.

In working these high pressures with great expansion the ordinary mode of packing the pistons was found unsatisfactory, and to overcome the difficulty the compound piston was devised. The prevalent scoring and cutting of engine cylinders was effectually remedied by the discovery of the compound metal, of which the packing rings are made, which requires no lubricating material. Many cylinders fitted with piston rings made of this metal have been several years at work, and have been often examined, the cylinders showing no signs of wear, the wear takes place on the rings only, which may be easily and inexpensively renewed as required, and experience has proved that with these pistons, the longer an engine is worked the more perfect does the surface of the cylinders become, and the less wear results to the packing rings. This metal for piston packing rings is composed of 5 parts tin and 15 parts copper, and has since been used by several other makers for ordinary engines with great success. When this metal is used, no oil or grease is required to lubricate the cylinders—a great advantage, particularly where the engines are fitted with surface condensers.

The high-pressure piston in the steamers Atacama and Coquimbo of the Pacific Steam Navigation Company were fitted with these packing rings, and it was reported by the superintendent engineer that the cylinders, which were previously rough and slightly grooved, were in the course of two or three voyages, or about 10,000 miles' run, brought up to a beautiful smooth surface, and had since kept in capital order, giving no trouble whatever. After having been once brought up to a smooth working surface, the packing rings did not wear the cylinders; the wear of the rings was also very slight, and the friction greatly reduced, and one-third of the lubrication necessary for cast-iron rings was found sufficient. In the torpedo vessels made for the French Government, Messrs. Thornycroft found these

packing rings for the engine pistons a great advantage, as there was no chance of the cylinders being scored; and they were enabled to run the two hours' trial easily, at the high speed of about 430 revolutions per minute, without using any oil or grease in the cylinders. In an engine at the Dorking Grey Stone Lime Company's works, the manager reported, after $2\frac{1}{2}$ years' use of these packing rings for the piston, that they required no grease of any kind, and worked the cylinders to a polished face and needed no looking to until worn out; a set of rings lasted about 100 days, working at the usual high steam pressure of 400 lbs. per square inch.

The surface condenser used is constructed of a number of vertical tubes in such a manner as to be absolutely tight, so as to insure that the condensing water inside the tubes shall not mix with the water from the condensed steam outside them. The tubes are $\frac{7}{8}$ in. internal and $1\frac{6}{16}$ in. external diameter, welded up at the top end and fixed securely in a tube plate at the bottom. These tubes are fitted with internal tubes, open at both ends, which are fixed in a division plate at the bottom, in order to cause the condensing water to circulate to their extreme ends.

A small still, worked by a steam coil, is used to distil water for replenishing any small waste that may take place in the feed supply. A duplicate apparatus forms part of the ordinary equipment of a sea-going vessel, to furnish steam from sea-water, for blowing the steam whistle, cooking, supplying distilled water for use of passengers and crew, and for all other purposes where distilled water is required.

In designing the machinery described, provision is made for passing any waste steam from the safety valves, etc., into the surface condenser, and the great strength of the boilers allows a margin of 100 lbs. per square inch or more to exist between the load on the safety valves and the pressure required to work the engines. When this system is fully carried out in steamships, the author would deem it quite safe, and more than ample for making good the waste of water from all sources, to provide, beyond the water in the boilers, a supply of fresh water in the proportion of 10 gallons per 24 hours per 100 indicated horse power. As an instance of the practical feasibility of carrying out the system of machinery that has been described, it may be stated that a boiler containing only 300 gallons, and an engine working at 250 lbs. pressure and 250 indicated horse power, were worked night and day continuously thirteen days (one Sunday excepted) without

requiring any addition to make good the waste of working, nor at the end of the trial was there any appreciable difference in the water level of the boiler.

In the indicator diagrams exhibited, the two upper diagrams are taken from the working of a pair of marine engines on this plan of 70 nominal horse power, and the coal consumed averaged 1·62 lbs. per indicated horse power per hour. In this case there was no vacuum and no low-pressure cylinder, and the terminal pressure was 21 lbs. per square inch above the atmosphere; the boiler pressure was 300 lbs. per inch. With the addition of a low-pressure cylinder and a vacuum, the author considers it may safely be estimated that this system, properly carried out, will realize an average duty of one horse power for each pound of coal per hour.

A NEW METHOD OF COMPRESSING AIR.

By Jos. P. Frizell, C. E., Boston.

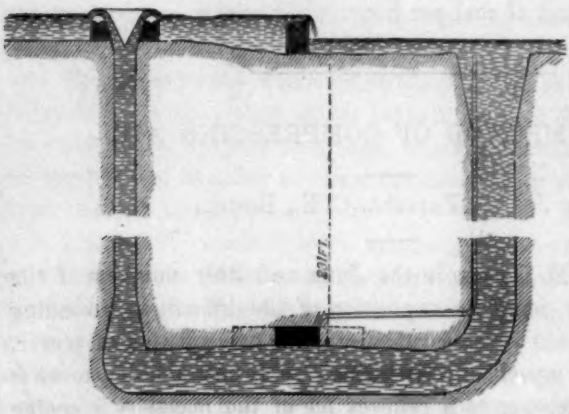
The memoir of M. Cornet in the June and July numbers of the Journal, is a very complete exposition of the difficulties attending the use of compressed air as a vehicle for the transmission of power. I am glad to avail myself of this memoir as an introduction to what follows, since a reference to it relieves me of the necessity of going into physical discussion in pointing out a method of compressing air entirely free from the sources of loss arising from the development of heat, as well as others inherent in the use of mechanism.

Bubbles of air rise in still water with a very moderate velocity; the velocity depends upon the size of the bubble, increasing as the latter increases, and this again depends upon the size of the orifice through which the air enters the water. From observations of my own, I find that bubbles issuing from an orifice of one-fourth of an inch diameter, do not rise faster than 15 inches per second in water of less than 20 feet depth. From depths of 50 feet the velocity is less than one foot per second. I assume in what follows that the average velocity in rising from depths of over 50 feet is one foot per second. This does not apply to great masses of air, such as rise when a diver is at work,

but to bubbles controlled in size by suitable orifices. Conversely, air drawn into a current of water which moves straight downwards with a velocity of more than 15 inches per second, will be carried down and subjected to a pressure corresponding to the depth attained. At the depth corresponding to the desired pressure, suppose the channel to take a horizontal direction. The air will here, in the course of a few seconds, rise to the top, and if it communicates at the summit with a suitable chamber, the air will enter it and accumulate under the desired pressure.

These considerations indicate the mode of transforming the power of a waterfall into compressed air, by the arrangement suggested in Figs. 1 and 2. The waterfall is supposed, as almost always occurs, to overlie a solid rock. At a point above the dam a vertical shaft is sunk to a depth corresponding to the pressure required. For a pressure of 100 lbs. per square inch the clear depth would be 231 feet. Thence a horizontal tunnel extends to a point below the dam, meeting another vertical shaft which reaches to the surface. The top of the tunnel is not exactly horizontal, but rises slightly from both ends toward the middle, at which point a chamber of sufficient capacity is excavated in the rock, whence the air is drawn off through a pipe. The whole forms thus an inverted siphon, through which the water flows with a velocity depending on the effective head. The length of the tunnel will be controlled by the necessity of placing the entrance to the chamber far enough from the descending branch to admit of the complete escape of the air bubbles.

Fig. 1.

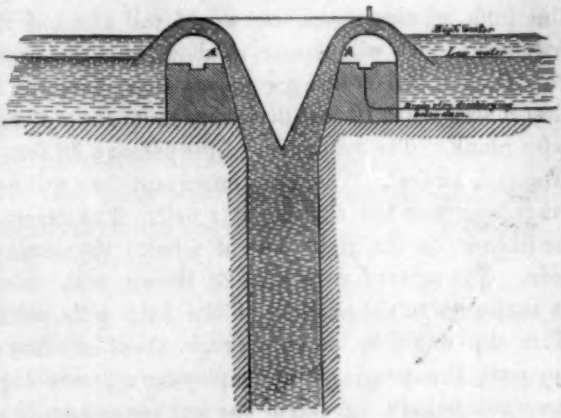


The effective head will depend on the quantity of air carried by the descending branch. This can be regulated at will, and furnishes the means of controlling the velocity.

Fig. 2 shows the method proposed for introducing the air bubbles into the descending column, though it is probable that a simpler method would disclose itself in the practical operation of the apparatus.

The entrance to the vertical shaft is surrounded by a bulkhead of masonry. Over this bulkhead the water is led in a covered channel whose bottom rises a little above the highest level of the water in the pond, forming a siphon. The arched space below the siphon and above the masonry, forms an accessible gallery, drained by a pipe which discharges below the dam. This gallery contains the cocks for regulating the admission of air. They are at *A* on the descending branch of the siphon channel, where its direction is nearly vertical. At this point the pressure within the siphon is less than that of the external air, and the latter will flow in through any opening; even should the external water rise as high as the top of the gallery, the internal pressure will be negative at *A*. This is evident from the fact that the flow does not take place by the direct action of gravity, but by the excess of the atmospheric pressure over that in the channel near the entrance. Were the entrance to the siphon closed by a gate, the pressure inside would be less than that of the atmosphere to a point as low as the surface of the water below the dam. It is apparent that the siphon channel extends entirely around the vertical shaft, except at one point, where it is interrupted by an entrance to the gallery. It is provided with a pump, worked by air from the chamber, to produce the vacuum when required, and remove, from time to time, the small quantity of air eliminated from the water. By allowing the air to enter the summit of the siphon, the flow may be suspended at will.

Fig. 2.



We have here a mode of compressing air entirely free from the difficulty which forms the subject of M. Cornet's memoir. The temperature of a minute bubble of air enclosed in a mass of water during the entire period of compression, which, in the case assumed, will last as much as 24 seconds, cannot rise to any serious extent. Let us endeavor to ascertain approximately what the losses of power would be by this method, or, by how much the power stored up in the form of compressed air would fall short of the absolute power expended. We will assume, as above, that the air is to be compressed to a tension of 100 lbs. per square inch above the atmosphere. The descending shaft 10 feet diameter, square in section, and lined smooth with plank. The remainder of the passage 20 feet diameter, unlined. The fall, 10 feet. The most important loss will be one analogous to what occurs in the slipping of a belt. The descending column may be likened to the tight side of a belt; the ascending, to the loose side. The upward movement of the air with reference to the water is analogous to the slipping of the belt with reference to its drum. This slip would be, on an average, about one foot per second, which suggests the propriety of employing a pretty high velocity in the descending shaft. A slip of one foot per second, in a belt moving three feet per second, involves a loss of 33 per cent. of the power, while the same slip, in a belt moving 10 feet per second, implies a loss of only 10 per cent.

The loss of head in a channel 230 feet long, 10 feet diameter, with a velocity of 10 feet per second, would be, according to Darcy and Bazin's experiments on channels lined with wood, 0·478 ft. Nearly the same result is obtained by Darcy's formula for cast iron pipes coated with bitumen. The loss in the remainder of the c annel assumed, 400 feet long, 20 feet diameter, would be but 0·078 f t. The loss due to the several bends would be, by Weisbach's formula, 0·239 feet. It would appear necessary, at first sight, to allow the head due to a velocity of 10 feet per second in addition to the preceding, but this is not so. By a suitable enlargement of the discharging end of a submerged channel, a velocity can be obtained, much greater than that due the apparent head. The momentum of the water, as its velocity diminishes, is employed in diminishing the pressure of the ascending column, and thus increasing the effective head. Mr. Francis, in this manner, obtained a velocity of 10 feet per second

with an apparent head of only 0·3 feet. We will allow 0·4 feet for initial velocity. Putting the several losses together, we have:

Loss due to friction in descending shaft, . .	0·478 feet.
" " " remainder, . .	0·078 "
" " bends, ...	0·239 "
" " initial velocity, . . .	0·400 "
" " slip, being $\frac{1}{10}$ of the remaining head,	0·880 "
Total, ...	2·075 feet,

which is about 21 per cent.

Again, suppose the fall to be 3 feet, and other conditions the same as before. We should probably find it advisable to reduce the velocity to, say 5 feet per second in the descending shaft. The losses from friction, bends, etc., being nearly as the square of the velocity, would be about one-fourth of the above, viz.: . 0·30 feet.

Loss due to slip, $\frac{1}{8}$ of 2·7, . . .	0·54 "
Total, ...	0·84 feet,

which is about 28 per cent.

A fall of 2 feet could be utilized with a loss of not over 40 per cent. For a fall higher than 10 feet we could expect a higher degree of economy. With a fall of 20 feet, for instance, the computed losses are 15·4 per cent. For practical purposes it would of course require extended calculations to find the most economical pressure of air and velocity of water for any given case.

We may affirm, then, that so far as our knowledge of hydraulics enables us to foresee the working of this apparatus, it will, on falls of 3 feet and upwards, convert 70 to 85 per cent. of the absolute power of the water into compressed air, at a tension of 100 lbs. per square inch. This transformation, by existing methods, involves the following losses:

1. An average loss of $\frac{1}{4}$ in the water-wheels.

2. A loss of at least $\frac{1}{8}$ in operating the mechanism of the air-compressor.

3. Of the power actually employed in compressing the air at 100 lbs., $\frac{1}{4}$ is lost, according to M. Cornet's calculations, in overcoming the resistance due to the development of heat. We have, therefore, for the part of the power actually realized in the form of compressed air, $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{3}{4} = \frac{63}{128}$, or a little less than $\frac{1}{2}$.

A comparison of the cost of this system with that of the water-wheels, canals, races, and mechanism, for accomplishing the same work, will show a result no less strikingly in its favor, not to mention its applicability to falls too low to be worth improving by the ordinary methods.

Its applicability to the much discussed problem of utilizing the tidal power will be at once apparent.

The loss of power due to the disappearance of heat during expansion in the air engine, attaches to this method, as to others. M. Cornet's method of dealing with this difficulty, by the injection of water in the form of spray, would undoubtedly be preferable in mines. In factories, a jet of high pressure steam would probably be preferable.

In factories, this difficulty is much more than counterbalanced by the advantage offered for the economical employment of heat. 1000 horse power in the form of compressed air, may be increased to 1400 by raising the temperature of the air 200 degrees F. The 400 additional horse power is secured at an expense not exceeding ½ lb. oal per hour, per horse power.

Whooping Cough.—It is some years since Letzerich affirmed that whooping cough was due to a special fungus. The assertion has been lately confirmed by the researches of Tschamer. In the spittle of children who are suffering from the cough, there are little corpuscles, about the size of a pin's head, of a white or yellowish color, which pass through a series of characteristic changes, and which seem to be identical with fungi which are found on the peel of oranges, apples and some other fruits. By inoculating rabbits with these fruit fungi, and by causing men to inhale them, Tschamer produced convulsive coughs of many days' duration, with all the characteristics of whooping cough.—*Jahrb. f. Kinderheilcunde; Les Mondes.* C.

Sensitive Thermometer.—An optician in Paris has constructed a new metallic thermometer. The expansion of a small sheet of platinized silver is amplified by a system of levers, and the motion communicated to a dial needle, as in Breguet's thermometer. It has been very satisfactorily tested in the new balloon, "Ville de Paris.' —*Nature.* C.

THE ELECTRIC REGISTER AND KOENIG'S TUNING FORKS.

By LeR. C. Cooley, Ph. D.

The application of electricity to register rapid vibrations, was first made in 1868.[i] In that year I succeeded in obtaining a direct registry of the vibrations for all the intervals of three successive octaves on a piano.[ii] The wire was made to open and close a battery circuit in which an electro-magnet was placed, whose armature carried a stile, and, responding to the alternate efforts of the electric pulses and an opposing spring, flew back and forth in unison with the vibrating wire, and left a dot for each vibration on a strip of moving paper.

But the motions of an armature are too sluggish to keep pace with vibrations beyond a certain degree of rapidity; hence, the electro-magnetic method was speedily abandoned, and an electro-chemical registry substituted. "The requirements are: first, a steady stream of electricity from a powerful battery; second, means by which the vibrating body may open and close this electric circuit; and third, a rapid motion of chemically-prepared paper, through which the electricity is passing."[iii]

In the instrument first constructed, a fine copper wire, forming one electrode of the battery, was rapidly drawn along over the surface of a strip of tin-foil forming the other. A strip of tissue paper moistened with acidulated water, lay upon the tin-foil; upon it the moving wire constantly pressed, and every electric pulse traversing the paper left a velvety black stain upon the foil beneath. The vibrations in any measured period of time were thus revealed in a series of dots easily counted and permanent in the highest degree; records made then, are as distinct to-day.

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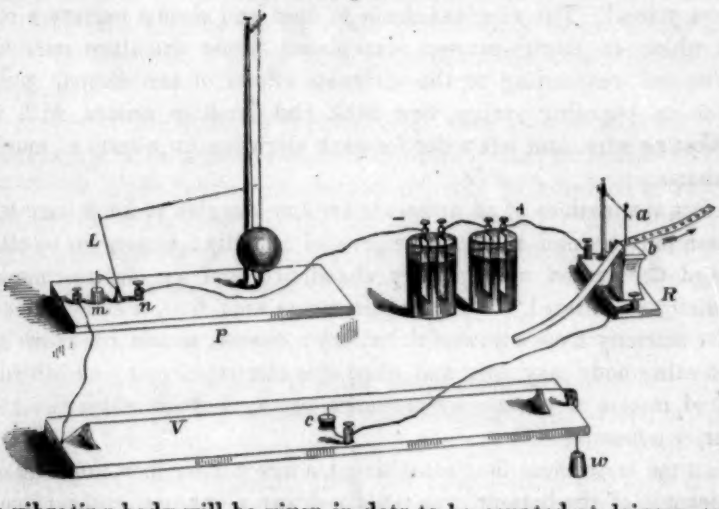
[i] *Proceedings of the Albany Institute*, Vol. i, p. 80. Journal of the Franklin Institute, Vol. lvii, p. 44.

[ii] Journal of the Franklin Institute, Vol. lvii, "Annual of Scientific Discovery," 1870, p. 162.

[iii] Cooley's "Text-Book of Natural Philosophy," p. 299. 1868.

With this instrument the laws of vibration were demonstrated by an almost unerring registry in several courses of lectures, yet certain difficulties in manipulation forbade the expectation that it would become generally useful. By changing the form of the apparatus and perfecting the method, however, the electro-chemical registry became, it seems to me, a matter of comparative ease, enabling one to study phenomena depending on rapidity of vibration, with the precision attained by actual count. Whenever the vibration is of amplitude sufficient to open and close an electric circuit, an *autograph* of

Fig. 1.



the vibrating body will be given in dots to be counted at leisure, and representing its rate within very narrow limits of error.

The accompanying cut represents the apparatus, arranged for the study of vibrating wires. The wire, *V*, a time measurer, *P*, and the register, *R*, are included in the same battery circuit. The wire is enabled to open and close the circuit by means of a fine steel needle, fixed to its middle point, beneath which is a cup of mercury, *c*, the surface of which is so nearly in contact with the point at rest, that every vibration of the wire will immerse it in the liquid metal. With good mercury, the needle being a substance not easily amalgamated and the surface of the mercury covered with alcohol to prevent oxidation, when once adjusted, little attention is needed afterward. Clean mercury may be substituted from time to time, and the needle may, if necessary, be cleaned by nitric acid.

For the measurement of time a pendulum is employed, which holds the circuit during the time of one beat. The arrangement for this purpose is represented at *P*. A slender fibre is fastened to the pendulum-rod, and thence reaches over to the upper end of a light bent lever, *L*. This lever moves freely, and is in conducting communication with a binding post, *n*. Beneath the lower end of the lever is a mercury cup, *m*, in metallic connection with another binding post. When the heavy pendulum is at rest, the weight of the lever keeps the fibre tense, and the mercury surface is so adjusted as to be exactly in contact with the point of the lever, a most vital adjustment, but one very easily made. It will be seen that the pendulum, when vibrating, must compel the lower end of the lever to be alternately in and out of the mercury during the exact time of one vibration.

The record of the vibrating wire is made in the register, *R*. A metallic point, *a*, presses upon a strip of chemically-prepared paper, which runs over a platinum surface, *b*. The pen and the platinum beneath, are each provided with a binding post, by which it can be made a part of the circuit. The paper may be drawn through by the hand of the operator, the more swiftly, as the vibrations are more rapid.

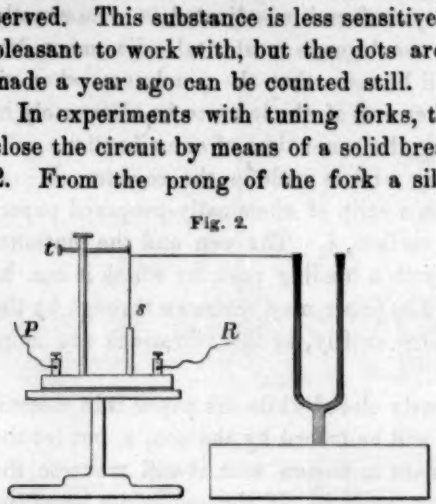
Let the circuit be continuously closed while the paper is in motion, and a continuous colored line will be traced by the pen, *a*, but let the wire vibrate, and electric pulses in unison with it will traverse the paper, leaving a series of dots instead. If the pendulum be at the same time in motion, the pulses can traverse the paper only while the lever, *L*, is in mercury, and hence a *group* of dots on the paper will represent the vibrations of the wire in the unit of time.

Various electrolytes may be used in the preparation of the paper. A strong solution of potassic iodide, mixed with a small proportion of starch, is very sensitive and easily managed. A strip of paper, strong, thin, with a smooth surface—strips cut from a sheet of book-paper of good quality, not over thick, are excellent—is *moistened* with this mixture. If too wet the dots will run together, if too dry the electricity will not traverse the paper; the strip should be moistened uniformly, but not wet. The record is written by a platinum pen, *a*, in dots of a reddish-brown color. Let it be at once washed in a stream of water, and the expected blue color takes the place of the brown; the fluid is at the same time washed from the paper, leaving the record in a condition to be more permanent.

Or, saturate the paper with a mixture of potassic ferrocyanide and nitric acid, made as follows: To four parts of a saturated cold solution of ferrocyanide, add one part of dilute nitric acid—one ounce acid to five of water. If the color of the mixture is decidedly blue, too much acid has been used; if no hint of blue appears, too little. With this fluid the pen, *a*, should be of copper, and the record consists of brownish-red dots of cupric ferrocyanide. Let the paper be washed to remove the excess of fluid, and the record may be preserved. This substance is less sensitive than the iodide, and not so pleasant to work with, but the dots are more permanent. Records made a year ago can be counted still.

In experiments with tuning forks, the vibrations may open and close the circuit by means of a solid break-circuit represented in Fig. 2. From the prong of the fork a silk fibre stretches across to a slender vertical spring, *s*, fixed at the lower end, while its upper end rests against the end of a set screw, *t*. The two surfaces in contact are platinized. The spring is slightly bent, and the set screw, pressing against its upper end, holds it in constant tension, and thus forbids it vibrating, except in unison with the fork. Every vibration of the prong is transmitted by the fibre, and compels the spring away from contact with the screw. Putting this break-circuit in place of the monocord, represented in Fig. 1, between the pendulum, *P*, and the register, *R*, the vibrations of the fork record themselves upon the moving paper.

Fig. 2.



Noticing that the accuracy of Koenig's tuning forks is questioned by Mr. Ellis (*Nature*, xvi, p. 85), I fancied that the testimony of this method would not be without interest. Seizing the earliest opportunity, therefore, I submitted the Ut_3 fork, bearing Koenig's monogram, to careful examination. The pendulum was accurately adjusted to hold the circuit one-half a second. The iodide-starch solution, with a battery of ten Bunsen's immersion cells, was used. Fifteen perfectly distinct and easily counted records were taken Every one of these autographs was found to consist of 128 dots, representing

128 complete vibrations per half second, 256 per second, or 512 according to the French notation.

But the instrument cannot record fractions of a vibration; on the other hand, no complete vibration can escape its registry. Evidently when the rate of the fork is any whole number of complete vibrations, that number must be recorded. Exactly 128 vibrations, for example, would yield invariably 128 dots; 129 would yield 129 dots invariably; but 128 and a fraction vibrations would yield 128 dots in some experiments, and 129 in others, and in this case the mean of several experiments would be a close approximation to the exact value of the fraction.

Now in the experiments with the Ut_3 fork the autograph *invariably* consisted of 128 dots, declaring the rate of the fork to be 256 complete vibrations per second, testifying to the exactness of Koenig's stamp.

ON THE DEVELOPMENT OF THE CHEMICAL ARTS, DURING THE LAST TEN YEARS.[1]

By Dr. A. W. Hofmann.

From the *Chemical News*.

[Continued from Vol. civ, page 119.]

Purification of Sulphuric Acid.—The sulphuric acid of commerce generally contains small quantities of lead, iron and arsenic, besides traces of selenium and thallium. It is only in some few of the arts that a pure acid is required on the large scale, *e. g.*, in the preparation of sulphate of soda free from iron for the plate glass manufacture. In most cases the iron and lead present in sulphuric acid do not interfere with its applications, and it can be freed from them, if needful, by simple distillation.

The removal of the arsenic is of greater importance, and for this purpose various methods have been proposed. H. A. Smith, in his

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[1] "Berichte über die Entwickelung der Chemischen Industrie Während des Letzten Jahrzehends."

work already quoted, has given detailed researches on the amount of arsenic acid in sulphur ores. He found in pyrites from:—

		Per Cent. Arsenic.
Spain,	Tharsis,	1·651
	Mason,	1·745
Belgium,		0·943
Westphalia,		1·878
Norway,		1·649
"	rich in sulphur,	1·708

On roasting, a part of the arsenic remains in the burnt ore, a part is deposited in the flues leading to the chambers; a further portion arrives into the acid, and passes along with this into the various products prepared with its aid, and is finally found in the sulphur recovered from alkali waste. According to Smith there is present in:—

	Per Cent. Arsenic.
Norwegian pyrites (hard kind),	1·649
" " burnt,	0·469
Sulphuric acid,	1·051
Flue dust before entering chambers,	46·360
Chamber mud,	1·857
Hydrochloric acid,	0·691
Salt-cake,	0·029
Alkali waste after lixiviation,	0·442
Soda,	0·000
Regenerated sulphur,	0·700

As to the removal of arsenic from sulphuric acid, several important observations have been made in the last few years. Bussy and Buignet[1] have examined the customary methods for the removal of this impurity, and pronounce them insufficient. The precipitation of the arsenic by means of sulphureted hydrogen or barium sulphide, is not complete, and the use of these means involves a considerable dilution of the acid. If we attempt to purify an impure acid by distillation, we obtain, under certain circumstances, a product free from arsenic, but in most cases an arseniferous acid passes over. Bussy and Buignet have ascertained the conditions in which a pure acid is obtained. They found that sulphuric acid containing arsenic in the form of arsenic acid can be purified by distillation, but not such as contains arsenious acid. Arsenic acid remains entirely in the residue, whilst arsenious acid passes over. Hence, Bussy and Buignet recom-

[1] Bussy and Buignet, *Dingl. Pol. Journ.*, clxii, 454.

mend to treat the commercial acid, which generally contains both arsenious and arsenic acid, with nitric acid, in order to peroxidize the arsenious acid. The acid is then mixed with a little sulphate of ammonia, in order to destroy nitrous acid, and distilled.

Büchner[i] has modified his original process for the removal of arsenic, which consisted in heating the sulphuric acid with hydrochloric acid. He has perceived that arsenic, when present in crude sulphuric acid as arsenic acid, cannot be removed by hydrochloric acid. To convert arsenic acid into arsenious acid, the crude acid to be purified must either be first heated with charcoal and then treated with hydrochloric acid, or the heating with charcoal and the treatment with hydrochloric acid are conducted simultaneously. As experience proves that the acid in most cases contains arsenic acid, the treatment with charcoal is to be recommended in all cases.

Blondlot[ii] converts the arsenious acid into arsenic acid prior to distillation, not by nitric acid, but by manganese peroxide, or the manganate of potash.

Lyte[iii] proposes to heat the raw acid first to 110° in a bowl with ¼ to ½ per cent. sulphuric acid, then to mix with chromate of potash, and finally to distil. The oxalic acid removes nitrous acid, and the chromic acid converts the arsenious acid into arsenic acid.

At Freiberg and Oker, sulphuric acid is purified by sulphureted hydrogen, and this process is particularly recommended, as the gas is not conducted into the liquid. The sulphuric acid is allowed to flow down a precipitating tower, fitted with prisms like a Gerstenhöfer furnace, but made of lead, and so arranged that an angle is always upwards and a side downwards. Into this tower enters from below a current of sulphureted hydrogen evolved from green pyrites and sulphuric acid. As the acid flows down in thin layers, the precipitation of the arsenic sulphide is very satisfactory.

Concentration of Sulphuric Acid.—The kinds of apparatus generally employed in sulphuric acid works for concentrating the chamber acid, are:—

1. Evaporation in leaden pans, standing upon cast-iron plates heated by the direct action of fire beneath the plates.

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[i] Bückner, *Bayer Kunst. u. Gewerb.*, 1864, 480.

[ii] Blondlot, *Comptes Rendus*, lviii, 76.

[iii] Lyte, *Chemical News*, x, 172.

2. The action of a reverberatory fire upon leaden pans, the edges of which have double sides and can be cooled by a current of water in order to prevent the lead from melting. Or the concentration is effected—
3. By steam.
4. By hot sulphurous acid.

In the application of the first-mentioned arrangement for concentration (open pans with direct fire), the author considers it advisable[i] to regulate the evaporation by the thermometer, as the lead is readily destroyed at a too elevated temperature.

At the commencement of the treatise just quoted, the author enters into an examination of the action of sulphuric acid upon lead, and states as the result of his observations that perfectly pure lead is more attacked during the concentration of sulphuric acid than a less pure metal. The same observation had been already made by Calvert and Johnson.[ii]

If the open pans employed in the evaporation are not made of too soft a lead, they last for a long time, at least if the concentration is conducted with due care.

Chandelon makes the judicious suggestion that the fire-gases from every arrangement for the concentration of sulphuric acid should be passed into a small separate chimney, since it is not possible to determine if a loss of acid is taking place when steam, hydrochloric acid gas, and the volatile products of the furnaces of a chemical works are all led into one large chimney.

The ordinary concentration in open pans is simple, and is still, therefore, principally in use, though not greatly to be recommended as far as repairs, consumption of fuel, and loss of acid are concerned.

The apparatus in which the flame plays directly over the surface of the acid was at one time widely used in England, and was first introduced into Germany at the Lüneburg Chemical Works. The furnaces last a long time without repairs, and consume little fuel, but are liable to the defect that an excessive temperature is often produced, when considerable quantities of sulphuric acid escape along with the products of combustion. On this account such furnaces

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[i] Hasenclever, *Ber. Chem. Ges.*, vi, 502.

[ii] Calvert and Johnson, *Comptes Rendus*, lvi, 140; *Dingl. Pol. Journ.*, 1863, 358.

have been abandoned in many places where they had been introduced.

The idea of concentrating sulphuric acid by the indirect action of steam, dates from the year 1865, and is due to Carlier, the manager of the Duisburg Chemical Works. After various experiments tried in this establishment, the evaporation, according to the report of F. Curtius, is conducted in wooden chests lined with lead, 4 metres in length and breadth. At the bottom of each chest are two leaden worms, each 45 metres in length, 3 centimetres in width inside, and 7 millimetres in thickness of metal. In order that the condensed water may flow off easily from the pipes, the bottom has the form of a blunted pyramid, the receptacle being 0·60 metre high in the middle, and only 0·3 at the ends. Both ends of the piping are in connection with the steam boiler, and can be shut off by means of cocks. The boiler is fixed lower than the concentration chests, which receive their supply of steam from a pipe leading from the dome. The pipes which allow the escape of the steam from the condensation chest, slope towards the steam space in the boiler, so as to permit a reflux of the condensed water into the latter. The action is intermittent. The chest is charged with acid at 1·5 sp. gr., and heated by steam till the sp. gr. rises to 1·7. The entire contents of the chest are then run into a wooden cistern lined with lead. In this reservoir there is a worm through which the chamber acid must pass on its way to the concentration chest, so that the latter is always fed with acid already warmed. The pressure of steam in the boiler amounts to three atmospheres, and an apparatus of the size given yields in twenty-four hours 5000 kilos. of acid at sp. gr. 1·7. The consumption of coal is 9 kilos. per every 100 kilos. of concentrated acid. The waste of lead amounts to 0·2 kilo. per ton of acid. The boiler only requires to be supplied with water to compensate for the escape through faulty joints. It is recommended to fence in the concentration chest with a wooden screen, to prevent the workmen from being injured by the hot acid scattered abroad if a steam pipe should burst.

Delplace observed at the Stolberg Works that the leaden steam pipes are principally attacked just where they plunge into the acid. The dust, which, even though slight, still gradually accumulates upon the pipes, causes by its capillary attraction the sulphuric acid to rise a few centimetres above the level of the liquid in the pan. This

acid is quickly concentrated by the steam, and thus rapidly occasions a strong corrosion of the lead. To meet this defect at the place where the pipe plunges into the acid, a leaden bell is blown on, opening upwards, and of rather larger diameter than the pipe itself. The outer leaden surface of the bell is still coated with a thin, damp layer of dust, but which is no longer heated by the steam.

Concentration by steam has been of late years widely adopted. No sulphuric acid is lost on account of the low temperature employed, and the process has the further advantages of cleanliness, of a very small consumption of coal, and of an important saving of labor.

(To be continued.)

Power of Electric Light.—Late experiments at St. Petersburg show that the power of the light may be increased by covering the carbon with a thin sheet of copper, and turning the cup towards the object to be illuminated. The most economical machine tried was that of Alteneck ,which, with a galvanized carbon of 10 mm. diameter, gave a maximum of 20,275, and a mean of 14,039 candles. The light was sufficient to make objects visible, for military purposes, at a distance of 3080 yards.—*Nature.* C.

Davyum: A New Metal.—Serge Kern announces his discovery in June last, of a new platinoid metal which he calls *davyum*, in honor of Sir Humphrey Davy. It is hard, silvery in lustre, malleable at red heat, readily soluble in aqua-regia and very feebly in boiling sulphuric acid, yielding a yellow precipitate with caustic potash. Sulphureted hydrogen, passed through a dilute solution of the chloride, yields a brown precipitate which becomes black upon drying. Potassic sulphocyanide, with the same solution, is colored red, and if the solution of davyum in KCyS is concentrated, a red precipitate is obtained. Sp. gr. 9·385 at 25° C. Kern thinks that in Mendelejeff's proposed classification of the elements, davyum is the hypothetical element placed between molybdenum and ruthenium, in which case its equivalent should be 100. It would then rank as the second confirmation of Mendelejeff's predictions, gallium having been the first. It is probably rare. The platiniferous sand does not contain more than ·00045 of davyum.—*Comptes Rendus.*

THE MISSISSIPPI RIVER IMPROVEMENTS.

Report of Captain Jas. B. Eads, to Mr. Julius S. Walsh, President of the South Pass Jetty Company.

Sir:—I have the honor to submit the following report upon the improvement of the South Pass of the Mississippi river:

Head of Pass.—The depth on the shoal in the river obstructing the entrance to South Pass was 14 ft., or 15 ft. at low water, when the jetties at the mouth were begun. Until late in October, 1876, this shoal remained almost intact, while the channel between the jetties, 12 miles distant, had rapidly improved; and although 20 ft. of water could be carried out of the pass to sea, this was not available for commerce, because of the shoal obstructing the entrance at its head.

To create a channel through this shoal, an additional system of works was designed and constructed at the head of the pass, including two submerged mattress sills, 2 ft. thick, 70 ft. wide, and each over 3000 ft. long. These were sunk on the bottom of the river, across the entrance to South-west Pass, and Pass à l'Outre. The South Pass discharges only about one-tenth of the entire volume flowing out of the three passes, and these sills were designed to prevent any enlargement of the two great passes, and thus hasten the scouring action on the bottom of the new channel into South Pass. They are laid in deep water, and 30 ft. of depth can be carried into both passes over them. They are well ballasted with stone, and soundings show that the bottom of the river is now level with the tops of the sills. They will, therefore, be likely to remain permanently in their present position. These sills diminish the entrance into these two great passes about 7000 square feet each.

The works at the pass actually embrace and control the entire discharge of the Mississippi river, which is at this point one mile and three-quarters wide. They, therefore, surpass in magnitude any similar works in the world. By them the wide funnel-shaped entrance to South Pass has been permanently converted into a narrower and

parallel one of 850 ft. in width. With the exception of the east dyke, they were commenced after the flood of 1876 had begun to subside, and in October had so far progressed as to control the flow into the pass. The use of dredge boats was then resorted to on the centre line between the parallel piers, within which the new channel has been formed, to hasten its development. These succeeded in cutting a narrow channel, of about 20 ft. in depth, through the shoal; but owing to the exceptionally low stage of the river at this time, the current was so feeble, that but little impression was made by the river through this dredged cut, and on the 1st of February it was not sufficiently widened to be available for vessels drawing over 15 ft., or 16 ft.

As the present depth of the bar at the mouth is no greater than it was when the works at the head were completed, nor indeed as great, the cause which interrupted the deepening that had previously proceeded with such remarkable regularity between the jetties, will be fully understood when the effect of the works at the head of the pass is considered.

Before the works there were built, the width, on the crest of the shoal, of the volume of water flowing into South Pass, measured transversely to the current, was 2400 ft. A concentration of this volume between parallel piers only 850 ft. apart, was relied on to secure the requisite depth of channel through the shoal, but this concentration (as will be seen by a chart of the works) could only be obtained by the erection of dams and piers, which greatly diminished the flow into the pass, and thus, for the time being, completely suspended the erosive action of the current between the jetties 12 miles below them.

During November, December, and January last, owing to the extremely low stage of the river, we were not justified in expecting scour through the works at the head of the pass, but were prepared to find deposits in the pass, and also between the jetties. To rob the pass, even temporarily, of so large a portion of its water, as was for the time barred out by our works at the head, necessarily reduced its current and caused considerable deposits to be thrown down during the time so much water was excluded.

Under these circumstances the rise in the river was looked for with considerable anxiety. It occurred about the 1st of February, and, as was expected, soon caused a rapid enlargement of the channel

through the shoal that had for so many years obstructed the entrance into the pass. Seventy thousand (70,000) cubic yards of sand were excavated by the current during the first seven days of February. Since that date, as the charts sent to you from time to time, show, the enlargement of the channel through the shoal has been going on with great rapidity, and upwards of 450,000 cubic yards have been excavated from it up to the present time.

The following table gives in detail the extent of its enlargement to May 23, 1877:

Date of Survey.	Least Width of 20 ft. channel. ft.	Least Width of 30 ft. channel. ft.
February 1, 1877,	30	
February 8, 1877,	250	
March 1, 1877,	360	
March 16, 1877,	370	
April 13, 1877,	415	30
May 6, 1877.	415	150
May 14, 1877,	420	170
May 23, 1877,	420	190

No detailed survey has been made of the head of the pass recently, but occasional soundings show continued enlargement. The works there will need no further expenditure this year, and the entire force employed in their construction has been discharged.

Although the February rise soon declined, and the next one did not come till the middle of March, the relief which the sudden enlargement of the entrance gave to the whole pass below, was readily observed in the channel between the jetties. The river water was heavily charged with sediment, but no new deposits were found in the jetty channel. During the month of February little or no change between the jetties occurred; the least depth on the bar was 20·04 ft., and the least width of the 20 ft. channel was 130 ft.

The Jetty Channel.—By the end of February the restoration of the full flow into the pass was felt in the jetty channel. Early in March a scour commenced in the upper portion of the jetty channel, and the enlargement of the channel very soon assumed remarkable dimensions. This enlargement has gradually extended down, and is now at a point 9000 ft. below the upper end of the east jetty. Its greatest depth on the 22d of June was 89 ft. at a point about one mile below the upper end of the jetty, and where the United States

Coast Survey map of 1875 shows a depth at that time of only 13 ft. The width of the 22 ft. channel through this 9000 ft. varies from 300 ft. to 500 ft. The 30 ft. channel is at one place 500 ft. wide. No scour occurs near either jetty, the deepening being confined to the central part of the channel.

To this extraordinary scour is to be attributed a slight deposit at the lower end of the jetties, which was observed to take place simultaneously with the scour in the upper portion of the channel. Careful estimates from a survey made in May, show, that since the end of March, 544,103 cubic yards had been scoured from between the jetties above station 80, while only 77,315 cubic yards of this amount have been deposited below that point near the sea ends of the jetties. The balance—466,788 cubic yards, or 86 per cent. of the whole scour—was completely transported into the gulf, and has been carried by the current far beyond the reach of our surveys.

Expressed in a more graphic form, the amount moved in 58 days from between the jetties, in a distance of 5000 ft. is equal to a mass of earth 900 ft. in length, 544 ft. in breadth, and 30 ft. high. This is equal to 9381 cubic yards removed each day.

When we consider that the volume of water, charged with this enormous quantity of sedimentary matter (in addition to what it had brought down from the river to the jetties), had, just before entering the gulf, to travel up a slope against the tides and winds, the fact, that only 14 per cent. of over half a million cubic yards of sand was dropped before it reached the end of the jetties, is one of the most encouraging features yet developed in connection with the jetty improvement.

On the 25th of April, the 22 ft. channel, 200 ft. wide, only extended uninterruptedly down between the jetties to station 68; or 6800 ft. Forty-eight days afterwards, June 12, it had been enlarged to 300 ft. in width, and then reached to station 90, or 2200 ft. further down than on April 25, thus averaging nearly 46 ft. per day in its progress to the sea. As the next payment from the United States will be due when a channel 22 ft. deep, by 200 ft. in width, is obtained through the jetties, the rate of enlargement since April 25, furnishes a basis on which to found a reasonable expectation as to the time when the next payment will be earned. The distance yet to be enlarged to 22 by 200 ft., and thus give a channel of that size through to deep water, was, on the 12th of June, 3000 ft. At

the rate of 46 ft. per day it should be through by the 16th of August. On the 20th of this month the 22 ft. channel had reached a point only 450 ft. from the deep water of the sea, but below station 90 it had only a width of 140 ft.

The Gulf in Front of South Pass.—A careful investigation was made last month of the condition of the gulf bottom, directly in front of the jetties, in order to ascertain what effect an almost uninterrupted scour of nearly two years' duration would produce. An area of 51·7 acres extending 2730 ft. seaward from the old crest of the bar, was covered with a large number of soundings accurately located by reference to the triangulation stations of the United States Coast Survey. The results of this examination were then compared with the chart of the coast survey of May, 1875, when the following results were ascertained.

The volume of water within this 51·7 acres, directly beyond the old crest of the bar, was:

In May, 1877, ...	2,387,849 cubic yards.
In May, 1875, . . .	1,970,849 " "

showing a gain of water within this area in front of the bar of 417,000 cubic yards, which is equivalent to an average increase of depth of very nearly 5 feet over the entire area investigated. In May, 1876, a similar examination had been made of the same area, showing an average gain in depth from May, 1875, to May, 1876, of 2·47 ft. From May, 1876, to May, 1877, the average gain in depth was 2·52 ft. These facts seem to clearly prove, that as the compact discharge from the jetties increases in depth by the erosion of the bar, the moving prism of river water compels the sea current, which is almost constantly passing under it on the outer slope of the bar in a direction transverse to the river's discharge, to scour out more room for itself under the outflowing volume from the jetties.

The apprehended re-formation of the bar in front of the jetties is shown by these facts to be unfounded; and it, together with the fear that the pass itself would deteriorate and fill up under the effect of the jetties, may be laid aside forever.

Condition of the Works.—The jetties are in excellent condition. The works have been constructed under the immediate supervision of Colonel James Andrews, who had, until the 1st of June, an unusually large force employed. The portions which had sunk by the compression of the mattresses owing to the deposition of sand and

other sedimentary matters in the mattress work, have been built up above high tide, entirely out to the sea ends of both jetties. The recommendations of the commission of the United States engineers, who, under orders of the Secretary of War, inspected the South Pass improvements last November, have been carefully observed, and the jetties have been well ballasted with stone, as suggested by them. The grant is being executed in perfect good faith, and the structures, as far as then completed, were acknowledged by this commission to be of the permanent and substantial character required by law.

The works have been thoroughly tested by some very severe storms since last December, but with the exception of the destruction of a few mattresses at the sea ends of the jetties, which had not been previously ballasted with stone, the damage done to them was very slight.

Some anxiety has been expressed as to the destructive effects of the teredo upon the willows and piles used in the construction of the works. Fears upon this point are groundless, for the reason that the teredo does not attack wood covered with sediment. The piles used in the work constitute no part of the permanent jetties. The core or interior of the jetties is formed of willow mattresses. These become completely filled with sediment so soon as the current ceases to pass through them. Their exposed surfaces are covered with stone ballast, which in like manner becomes filled with sediment, and this completely protects the willow work from the attack of the worm.

The various surveys referred to have been made under the supervision of Mr. E. L. Corthell, resident engineer, and Mr. Max E. Schmidt, chief assistant, aided by Mr. Webb and Mr. Morton, assistant engineers; to all of whom I am much indebted for prompt and intelligent co-operation.

Official Orders.—I take great pleasure in stating that the honorable Secretary of War has issued such orders as will permit the prompt delivery to me of the results of examinations and surveys made from time to time by Captain M. R. Brown, United States Engineer Corps, United States inspecting officer of the jetties. He has also directed Captain Brown to establish such regulations for the navigation of the pass, as will protect the tugs, barges, etc., of the jetty contractors from injury by the too rapid movement of the steamers using the pass.

Results.—Our works were begun two years ago in an unused outlet of the Mississippi river, and have necessarily disturbed the regimen governing the outflow to the sea of an enormous volume of water; but the theories upon which they were based have been fully vindicated by the results produced; and it is now manifest that entire and complete success will reward our labors. Among the prominent results developed by our operations, are the following:

1. The concentration of the water flowing across the sand bar at the mouth of the pass by the jetties, created a channel over 200 ft. wide, in no place less than 20 ft. deep, where only about 8 ft. previously existed.

2. The concentration of the water flowing over the shoal in the river, at the head of the pass, created a channel over 400 ft. wide, in no part of it less than 20 ft. deep, with the central part 30 ft. deep, where but 14 ft. to 15 ft. previously existed.

3. During the time in which a portion of the flow into the pass was interrupted by the works at its head, and the current consequently slackened, a temporary deposit took place in the pass and between the jetties.

4. The gradual restoration of the normal flow into the pass through the new channel at its head has already begun to enlarge the pass again, and has, since this restored flow commenced, removed from between the jetties within the past three months over half a million cubic yards of deposit, and given through more than half the length of the jetties a much larger and deeper channel than ever previously existed, the size of which is already throughout more than 2000 ft. 28 ft. by 300 ft., or that required to entitle us to the fifth payment from the United States, while many hundred feet of it exceed 30 ft. by 350 ft.

5. The gulf current athwart the jettied mouth of the pass effectually prevents the re-formation of the bar in advance of the jetties by deepening the outer slope of the bar, and sweeps away any such portion of the discharged sediment as the river current fails to carry to unknown distances seaward.

6. The Mississippi river at the head of the passes, where it has a width of over 9000 ft., is brought under complete control by our works, which are so designed as to enable us to increase or limit the discharge into our pass, if hereafter necessary, with but little additional outlay.